Hierarchical Model Predictive Control for Building Energy Management of Hybrid Systems

Amanda Abreu * **Romain Bourdais** * **Hérvé Guéguen** *

* *CentraleSupelec-IETR, Avenue de la Boulaie - CS 47601, F-35576 Cesson-Sévigné Cedex France (e-mails: Amanda.Abreu@supelec.fr, Romain.Bourdais@centralesupelec.fr, and Herve.Gueguen@centralesupelec.fr).*

Abstract: In this paper a two-layer controller is proposed to tackle the building energy management problem for hybrid systems at different levels of abstraction and different time scales. In the upper layer a relaxed long term energy allocation problem with a large decision time step is defined, taking into account the energy prices, the comfort requirements, and a global power constraint. The discrete decision variables are considered only in the lower layer, where the continuous global solution computed by the first optimization is projected into local mixed-integer programming (MIP) tracking problems with a shorter prediction horizon and a higher sampling rate. To fulfill the building global power constraint each load has a specific priority to access the available power, following a non-iterative priority algorithm.

Keywords: Hierarchical control, hybrid systems, building energy management systems.

1. INTRODUCTION

The main goal of most Building Energy Management Systems (BEMS) found in the literature is to design control strategies that are able to maximize the comfort of occupants, to minimize the energy consumption costs, and to certify the good operation of the appliances, see Shaikh et al. (2014). As pointed by the survey paper Thieblemont et al. (2017), a large number of research works are based in hierarchical concepts due their ability to decompose the original energy management problem into hierarchically well-defined simpler subtasks, to consider distinct dynamics at different levels of abstraction, and to make it easier to define specific policies/roles for each one of the control levels, see some examples in Schirrer et al. (2016), and Mayer et al. (2017). In this paper, we focused in a particular type of hierarchical structure that implements multi-time scale problems to manage a set of different sub-systems, with a higher long-term scheduling optimization layer to compute overall tendencies with a large time step, and a lower layer that performs control decisions in a fast time-scale.

Regarding the literature of multi-time scale structures for BEMS, one relevant challenge still under-investigated is the management of devices with discrete control variables in systems with strict global power constraints. Appliances that are naturally present in most building scenarios such as heat pumps, stratified water tanks, ON/OFF valves, etc.; submit the final correspondent optimization problem to local integer/binary hard power constraints. In Lefort et al. (2013), for example, a scheduling layer periodically communicates to a piloting layer the optimal trajectories and the maximal amount of energy that should be consumed for each controllable device, but no integer constraints were treated. In Beaudin et al. (2014), a mixed-integer programming (MIP) scheduling algorithm is formulated, where the energy states of each device and the peak power consumption related to the long time period are transmitted to the lower level as set-points. Although, the information exchanged between the layers concerns only the continuous control devices.

To support discrete control appliances the multi-time scale architecture need to directly consider discrete decision variables. But the introduction of this kind of variable in the optimization problem of the upper layer would have no practical sense, since the control signals are piece-wise constant between two large time steps. Hence, the purpose of the higher layer is limited to the allocation of an average amount of energy among the sub-systems considering a global and long term relaxed optimization problem; raising important questions about the projection of the upper layer relaxed results into the lower layer optimization and the respect of the original discrete constraints.

Another essential feature that is also poorly contemplated is the fact that the building consumption must remain below a strict power threshold specified by contract with the electricity supply company. The authors of Yu et al. (2013), for example, proposed a stochastic MIP multi-time scale energy management problem in which the utility may restrict the power demand, but the global constraint was considered only in the upper slow time scale layer, where the integer local power constrains were relaxed.

In this context, the current paper is motivated by the necessity to address a general multi-time scale energy management problem for residential buildings with ON/OFF loads and global power constraints that are able to 1) Properly project the continuous solution computed by the relaxed problem of the higher layer into the discrete set of control variables of the lower layer. 2) Guarantee the hard global power constraint in the fast optimization time-scale. More specifically, we combine centralized and distributed approaches to define a two-layer hierarchical control structure that offers a balanced trade-off between optimality and computation time.

The remaining of this paper is organized as follows. In section 2, the building management scenario is defined and the control objective is formulated. In section 3, the proposed hierarchical approach is presented. Finally, in section 4, the hierarchical approach are applied to manage a group of ON/OFF loads and its performance is compared with the pure centralized and distributed benchmark solutions.

2. PROBLEM DESCRIPTION

2.1 Model and constraints

The hierarchical approach developed in this paper aims to manage a set of electric devices governed by discrete control variables in a residential building scenario where the power consumption is strictly limited by the public supply company. In order to select the optimal control actions that meet the objectives of the energy management system and respect the operation constraints, the model of each device as well as occupancy predictions and price profiles are used to predict the future behavior of the system.

Two types of power constraints are distinguished: global power constraints and local power constraints. The first ones have to be considered to ensure that electricity demand does not exceed the capacity of the utility company or in some cases the local production restrictions. The second ones refer to the electric limitation of the equipment and are necessary to prevent the appliances from damage caused by excess current. In order to study important effects of the introduction of these constraints, we focused on the management of ON/OFF controllable appliances. It means that the loads are allowed to have a discontinuous operation during the optimization period and the power consumption is controlled by binary decision variables. In this conditions, we assume that:

- The controlled system is composed by a set of electric devices denoted by $\mathbf{M} = \{1 : \mathrm{M}\}$, with $\mathrm{M} \in \mathbb{Z}^*$. The dynamics of each sub-system m, with $m \in \mathbf{M}$, is given by the following discrete-time model:
$$\begin{cases} x_{m,k+1} = f_m(x_{m,k}, u_{m,k}) \\ y_{m,k} = g_m^{\mathrm{y}}(x_{m,k}) \\ p_{m,k} = g_m^{\mathrm{p}}(u_{m,k}) \end{cases} \quad (1)$$
Where $k \in \mathbb{Z}^0$ is the time instant index, $x_{m,k+1} \in \mathbb{N}^{n_x}$ is the state vector, $y_{m,k} \in \mathbb{N}^{n_y}$ is the output vector, and $p_{m,k} \in \mathbb{N}^{n_p}$ is the power consumption vector.
- The input vector $u_{m,k}$ contains the discrete manipulated variables that are going to be optimized by the controller, with $u_{m,k} \in \mathbf{U}_m^{\mathrm{d}}$. The lower and the upper bounds of the control variable set are represented by $u_{\mathrm{m}}^{\min}$ and $u_{\mathrm{m}}^{\max}$.
- The building is connected to the electric power grid and the maximum power supplied is represented by $P_k^{\max}$, with $P_k^{\max} \geq 0$. The global power consumption is restrained by:
$$\sum_{m=1}^{\mathrm{M}} p_{m,k} \leq \mathrm{P}_k^{\max} \quad (2)$$
- The cost of the energy consumed from the grid is a time varying function represented by E_{k}.

2.2 Control objective

The objective of the building energy management system (BEMS) is:

- to minimize the energy consumption cost;
- to maximize the comfort of the occupants;
- to guarantee the global and the local power constraints.

The control goals are considered in a multi-objective criteria in order to find a compromise solution between the energy consumption cost and the discomfort of the users regarding the respect of the chosen operating set-points, represented respectively by J_{eco} and J_{dis}. The optimization is carried out in a finite prediction horizon Δh and a sampling time Δk, with $\mathrm{N} = \Delta\mathrm{h} / \Delta\mathrm{k}$ and $\mathrm{N} \in \mathbb{N}^*$. The correspondent discrete problem solved at time $t = \mathrm{k_t}\Delta\mathrm{k}$, with $\mathrm{k_t} \in \mathbb{N}^*$, is formulated as follows:

$$\min_{\hat{\mathbf{u}}_{\mathbf{M},\mathrm{H}}} \sum_{m\in\mathbf{M}} \sum_{k\in\mathbf{H}} J_{\mathrm{eco}}(\hat{p}_{m,k}, E_k) + J_{\mathrm{dis}}(\hat{y}_{m,k+1}, W_{m,k+1}^{y})$$
$$\text{subject to}$$
$$\forall \ m \in \mathbf{M} \ \mathit{and} \ \forall \ k \in \mathbf{H}$$
$$\begin{cases} i. \ \hat{x}_{m,k+1} = f_{\mathrm{m}}(\hat{x}_{m,k}, \hat{u}_{m,k}) \\ ii. \ \hat{y}_{m,k+1} = g_m^{\mathrm{y}}(\hat{x}_{m,k+1}) \\ iii. \ \hat{p}_{m,k} = g_m^{\mathrm{p}}(\hat{u}_{m,k}) \end{cases}$$
$$iv. \ \sum_{m=1}^{M} \hat{p}_{m,k} \leq P_k^{\max}$$
$$v. \ \hat{u}_{m,k} \in \mathbf{U}_m^d$$
$$vi. \ \hat{x}_{m,k_t} = x_{m,k_t} \quad (3)$$

The decision variables vector is composed by $\hat{\mathbf{u}}_{\mathbf{M},\mathrm{H}}$, which is the set of optimal control moves over the optimization horizon $\mathbf{H}$ for all $m \in \mathbf{M}$, with $\mathbf{H} = \{\mathrm{k_t}, \mathrm{k_t}+1, ..., \mathrm{k_t}+\mathrm{N}-1\}$. The desired operation set-points given by the users are represented by $W_{m,k+1}^{y}$. The discrete functions (i), (ii), and (iii) are used to predict the system behavior. The predictions for the state, the output and the power consumption are represented by $\hat{x}_{m,k+1}$, $\hat{y}_{m,k+1}$ and $\hat{p}_{m,k}$, respectively. The global power constraint and the local input constraints are defined respectively by (iv) and (v), and the initial state condition by (vi).

3. HIERARCHICAL CONTROL STRUCTURE

In the literature, similar optimization problems as the one depicted in equations 3 are solved using centralized and distributed approaches, see Haider et al. (2016). The first ones can ensure an optimal solution since the global MIP problem, including global constraints and all coupled variables, is solved in a single central controller. However, some shortcomings with respect to computational time, privacy, and reliability issues might turn the centralized approach prohibitive for real time applications in large

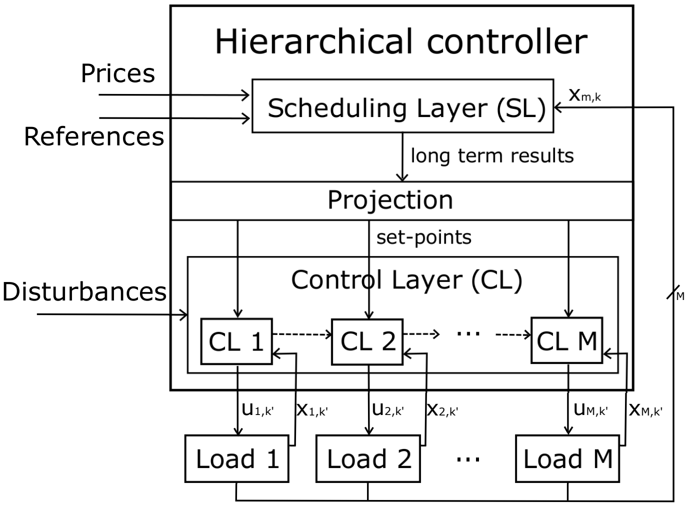

Fig. 1. Two-time scale Hierarchical Control Structure.

buildings. On the other hand, distributed strategies tries to break the global energy management problem into local smaller controllers, returning a sub-optimal but less computational demanding solution.

The idea of the hierarchical strategy formulated in this paper is to combine both strategies in order to offer balanced trade-off between optimality and computation time. The hierarchical approach is able to approximate the solution of the centralized MIP optimization problem through a modular architecture, where an upper layer operates to allocate the energy to all sub-system over a global point of view and a lower layer to respect the power constraints while following the local energy allocation tendencies. The computational complexity of the final control is kept small because the nonlinearities are considered only in the individual optimization problems of the lower layer, where the number of decision variables is reduced. To prevent communication issues, minimal information is exchanged between the global and the local controllers and a non-iterative procedure is used to reach coordination of the local controllers.

The hierarchical control structure is schematically represented in Fig. 1. The upper layer is denoted the Scheduling Layer (SL) and the lower one the Control Layer (CL). SL and CL operate at different sampling times represented by $\Delta \mathrm{k}_{\mathrm{SL}}$ and $\Delta \mathrm{k}_{\mathrm{CL}}$, and over different prediction horizons, denoted by $\Delta \mathrm{h}_{\mathrm{SL}}$ and $\Delta \mathrm{h}_{\mathrm{CL}}$, respectively, with $\Delta \mathrm{k}_{\mathrm{SL}} > \Delta \mathrm{k}_{\mathrm{CL}}$ and $\Delta \mathrm{h}_{\mathrm{SL}} > \Delta \mathrm{h}_{\mathrm{CL}}$. k and k' are the optimization time steps of SL and CL, respectively.

3.1 Scheduling layer

The main purpose of SL is to assign an average amount of energy among the sub-systems considering the global constraints and the slow varying behaviors (daily price predictions, daily reference profiles, thermal dynamics, etc.). To do that, SL implements the centralized energy allocation problem defined in equations 3 over a long time prediction horizon $\Delta \mathrm{h}_{\mathrm{SL}}$ and at a long time control step $\Delta \mathrm{k}_{\mathrm{SL}}$, with $\Delta \mathrm{h} = \Delta \mathrm{h}_{\mathrm{SL}}$ and $\Delta \mathrm{k} = \Delta \mathrm{k}_{\mathrm{SL}}$. In order to properly consider the energy consumption of the ON/OFF devices over the long time decision step, the discrete control variables are relaxed, redefining the local input constraints 3(v) as:

$$\begin{aligned} &\hat{u}_{m,k} \in \mathbf{U}^{\mathrm{c}}_{m} \\ &\text{with } \mathbf{U}^{\mathrm{c}}_{m} = \{u : u \subset \mathbb{N}^{n_u}, u^{\min}_{\mathrm{m}} \le u \le u^{\max}_{\mathrm{m}}\} \end{aligned} \tag{4}$$

Thanks to this relaxation, the computation effort needed to find the global solution is reduced but the resulting energy tendencies calculated as optimal by the higher layer are intermediate quantities that still need to be projected in a discrete set in order to respect the nonlinear constraints. For this purpose, the optimal output trajectory vector computed by SL for each one of the m loads, represented by $\hat{\mathbf{y}}_{\mathrm{m},\mathbf{H}_l}$, is transmitted to CL to be used as a reference to build the set-point vectors given to the local regulators. The details on the conversion of the smaller resolution results of the upper layer into the higher resolution set-points of the lower layer can be found in our previous paper Abreu et al. (2017).

3.2 Control layer

CL implements a dynamic tracking layer over a short prediction horizon $\Delta \mathrm{h}'$ and short control step $\Delta \mathrm{k}'$. In order to respect the integer constraints of the original problem, the variables relaxed by the centralized optimization implemented by SL must be considered into the optimization problem of CL, generating the necessity to define a MIP problem. However, as highlighted by Tsui and Chan (2012), the computation of mixed-integers problems for large scale systems can become prohibitive in a centralized approach since the time required to find a solution grows exponentially with the size of the system. So, to limit the mathematical complexity of the optimization process, CL is composed of M independent regulators that implement individual dynamic tracking problems performed in a distributed fashion. The local optimizations are carried out in a short finite prediction horizon $\Delta \mathrm{h}' = \Delta \mathrm{h}_{\mathrm{CL}}$ and a sampling time $\Delta \mathrm{k}' = \Delta \mathrm{k}_{\mathrm{CL}}$, with $\mathrm{N}' = \Delta \mathrm{h}' / \Delta \mathrm{k}'$ and $\mathrm{N}' \in \mathbb{N}'^{*}$. At time $t = k'_t * \Delta \mathrm{k}'$, for each m load, with $m \in \mathbf{M}$, the local MIP problem that has to be solved is formulated as follows:

$$\begin{aligned} &\min_{\hat{\mathbf{u}}_{\mathbf{m},\mathrm{H}'}} \sum_{k' \in \mathbf{H}'} J_{\mathrm{dis}}(\hat{y}_{m,k'+1}, WS^{y}_{m,k'+1}) \\ &\qquad \text{subject to} \\ &\forall \ k' \in \mathbf{H}' \\ &\qquad \begin{cases} i. \ \hat{x}_{m,k'+1} = f_{\mathrm{m}}(\hat{x}_{m,k'}, \hat{u}_{m,k'}) \\ ii. \ \hat{y}_{m,k'+1} = g^{\mathrm{y}}_{m}(\hat{x}_{m,k'+1}) \\ iii. \ \hat{p}_{m,k'} = g^{\mathrm{p}}_{m}(\hat{u}_{m,k'}) \end{cases} \\ &\qquad\ \ iv. \ \hat{p}_{m,k'} \le p^{\max,\mathrm{up}}_{m,k'} \\ &\qquad\ \ v. \ \hat{u}_{m,k'} \in \mathbf{U}^{d}_{m} \\ &\qquad\ \ vi. \ \hat{x}_{m,k'_t} = x_{m,k'_t} \end{aligned} \tag{5}$$

The decision variables vector is composed by $\hat{\mathbf{u}}_{\mathbf{m},\mathrm{H}'}$, which is the set of optimal control moves over the optimization horizon $\mathbf{H}'$, with $\mathbf{H}' = \{\mathrm{k}'_{\mathrm{t}}, \mathrm{k}'_{\mathrm{t}}+1, ..., \mathrm{k}'_{\mathrm{t}} + \mathrm{N}' - 1\}$. As explained before, the set-point vector $WS^{y}_{m,k'+1}$ is created taking into account the SL optimization results. The discrete functions (i), (ii), and (iii) are used to predict the short term system behavior. The short term predictions for the state, the output and the power consumption are represented by $\hat{x}_{m,k'+1}$, $\hat{y}_{m,k'+1}$, and $\hat{p}_{m,k'}$, respectively. The local integer input constraint and the initial state condition are defined by (v) and (vi), respectively. Finally, (iv) represents a local power constraint that is updated

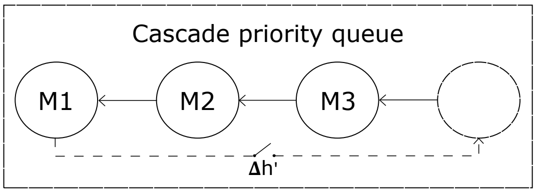

Fig. 2. Evolution of the load priority queue considering the cascade coordination procedure, where M1, M2, and M3 represent the execution of the local CL optimization problems.

for each individual controller following a cascade priority coordination procedure explained in the next sub-section.

3.3 Cascade priority coordination procedure

The global power constraint of the original optimization problem, represented by $3(iv)$, is a shared variable among all individual regulators. Hence, due the decentralized nature of CL, some kind of cooperation need to be applied to guarantee that the final integer control variables computed at each short sampling time respect the global maximum consumption threshold. In the literature of distributed control, negotiation protocols and iterative procedures are used to reach coordination. However, until now, no operational mechanism is universally successful in solving distributed optimization problems when the control variables are discrete, see Luo et al. (2017). In most cases, successive iterations may be need to converge to the optimal solution leading to communication and time issues.

For this reason, we choose to implement a non-iterative algorithm that applies a cascade priority coordination procedure. It means that the local optimization problems are solved sequentially respecting a pre-defined priority queue. Plus, at each new local optimization the value of global available power $p^{\max,\text{up}}_{m,k'}$ is updated in order to deduct the power already used by the loads that have bigger priority, with:

$$p^{\max,\text{up}}_{m,k'} = \text{P}^{\max}_{k'} - \sum_{m_p \in \mathbf{M}_\text{m}} \hat{p}_{m_p,k'} \tag{6}$$

Where $\text{P}^{\max}_{k'}$ represents the global power constraint and $\mathbf{M}_\text{m} \subset \mathbf{M}$ the set of loads that have bigger priority than the load m. This synchronization in the execution of the local problems can guarantee that the original global power constraint is respected and also requires little communication efforts since only the current available power is exchanged to the local controllers. Some literature works proposed to define the load priority list according to the users preferences or to the type of appliance, see some examples in the distributed strategies of Pipattanasomporn et al. (2012) and Liu et al. (2012). However, the idea of the present work is to try to find a solution as close as possible to the centralized results, where the priority among the sub-system is not explicit, but completely integrated in the multi-objective cost function. Therefore, to avoid a biased partition of the available power, the elements of the priority queue are circularly shifted for each short optimization horizon $\Delta\text{h}'$, giving to different loads the opportunity to have the higher priority at least once during the long optimization horizon Δh. The evolution of the cascade priority queue is represented in Figure 2.

4. SIMULATION RESULTS

To analyze the performance of the hierarchical strategy proposed in this paper, lets consider a building system composed of a set three identical domestic hot water tanks, with $\mathbf{M} = 1, 2, 3$. The loads have an ON/OFF control with $u_{m,k} \in \mathbf{U}^\text{d}_m$, $u^{\min}_\text{m} = 0\text{kW}$, and $u^{\max}_\text{m} = 4\text{kW}$, $\forall\ m \in \mathbf{M}$. The global power constraint is specified as $P^{\max}_k = 8kW$, meaning that a maximum of 2 water tanks can be On at the same time. The simulations are carried over a 4-hour period and the control signals are sent to the loads every 10 minutes. The energy daily price profile and the external temperature, which are considered as constants during the studied period, are predicted with no errors. Four different control approaches are implemented in a open loop simulation: the Centralized (C), the Decentralized with Fixed Priority (DFP), the Decentralized with Cascade Priority (DCP), and the Hierarchical (H):

Table 1. Configuration parameters for the strategies C, DFP, DCP and H.

Strategy	Layer	Horizon	Time step	Priority
C	-	$4h$	$10min$	-
DFP	-	$4h$	$10min$	Fixed
DCP	-	$30min$	$10min$	Cascade
H	SL	$4h$	$30min$	-
	CL	$30min$	$10min$	Cascade

- C: In C, the optimization problem represented in equations 3 is solved over a long prediction horizon $\Delta\text{h} = 4h$ and at a short control time step $\Delta\text{k} = 10min$. This approach is used as a benchmark in order to have an idea of the best possible performance.
- DFP: DFP implements a distributed control strategy composed of M independent regulators that locally minimizes the multi-objective criteria $\sum_{k \in \mathbf{H}}(J_{eco} + J_{dis})$. The generated optimization problem is similar to the one described in equations 3, except by the fact that each individual regulator minimizes only the local criteria. In this case, $\Delta\text{h} = 4h$ and $\Delta\text{k} = 10min$, and the local problems are solved sequentially respecting a fixed priority queue. The value correspondent to global power constraint is updated at each new optimization to consider the energy already consumed by the loads with bigger priority.
- DCP: As DFP, it is also composed of M independent regulators that implements local multi-objective optimization problems. However, $\Delta\text{h} = 30min$ and $\Delta\text{k} = 10min$. Hence, to provide the open loop results for the 4-hour simulation, one local optimization problem is built for each period of $30min$ (eight in total). The local problems are solved following the cascade priority coordination procedure explained in section 3.3.
- H: H applies the hierarchical strategy proposed in this paper, with $\Delta\text{h} = 4h$, $\Delta\text{k} = 30min$, $\Delta\text{h}' = 30min$, and $\Delta\text{k}' = 10min$.

All strategies are implemented using MATLAB R2016a, YALMIP (see Lofberg (2004)), and MOSEK 8.0.0.57. Their configuration is summarized in Table 1.

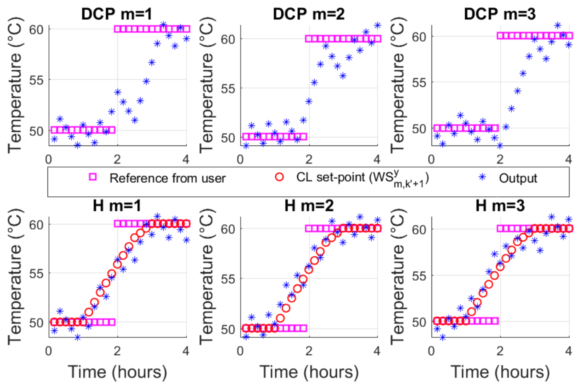

Fig. 3. Comparison between the output behavior of DCP and H.

4.1 Performance indicators

To compare the four strategies previously defined, three different performance indicators are considered:

- The value of the local multi-objective cost function represented by $J_m, \forall\ m \in \mathbf{M}$:

$$J_m = \sum_{k \in \mathbf{H}_p} J_{\text{eco}}(p_{m,k}, E_k) + J_{\text{dis}}(y_{m,k+1}, W^y_{m,k+1}) \tag{7}$$

- The value of the global multi-objective cost function represented by J_g:

$$J_g = \sum_{m \in \mathbf{M}} J_m \tag{8}$$

- The computation time to find the open loop solution represented by T_g.

The performance indicators are calculated a posteriori for each discrete time step k_p and over a 4-hour simulation period. In this case, $\Delta\mathrm{k_p} = 10min$, $\Delta\mathrm{h_p} = 4h$, $\mathrm{N} = \Delta\mathrm{h_p} / \Delta\mathrm{k_p}$, and $\mathbf{H}_p = \{\mathrm{k_p}, \mathrm{k_p}+1, ..., \mathrm{k_p}+\mathrm{N}-1\}$. Note that the lower the index J_g, the better the global performance of the respective approach. In terms of cost function, the main objective is to find the strategy that approximates the best the solution achieved with the Centralized approach. Hence, to facilitate the comparison, the normalized values of J_m and J_g with respect to the Centralized solution are represented by $\bar{J}_m$ and $\bar{J}_g$.

4.2 Energy cost and comfort deviation

To analyze the energy cost and the comfort deviation by means of J_m and J_g, 10 open loop simulations with different values of hot water drawing-off speed (between $5L/h$ and $40L/h$) are performed considering $\mathrm{P}_k^{\max} = 8kW$. The average value, the minimum value, the maximum value, and the standard deviation of $\bar{J}_g$ for each strategy are represented by $\bar{J}_g^{mean}$, $\bar{J}_g^{min}$, $\bar{J}_g^{max}$, and $\bar{J}_g^{std}$, respectively. Table 2 shows that the values of J_g are in average 6% bigger than the benchmark results when using the hierarchical strategy H, while 36% bigger when using DCP. Moreover $\bar{J}_g^{std}$ has its lower value when H is implemented, meaning that $\bar{J}_g$ slightly differs for the different simulations. As evidenced, H over-performs the distributed approaches DFP and DCP regarding the global cost function, approximating the solution the best to the Centralized results. The

Table 2. Average value, minimum value, maximum value, and standard deviation of $\bar{J}_g$.

Strategy	$\bar{J}_g^{mean}$	$\bar{J}_g^{min}$	$\bar{J}_g^{max}$	$\bar{J}_g^{std}$
C	100.00	100.00	100.00	0.00
DFP	136.17	107.23	165.74	22.04
DCP	133.54	116.46	155.25	12.38
H	106.12	100.45	115.70	5.11

comparison of H and DCP is particularly interesting due the fact that the main difference between them is the presence of the global upper layer that implements the relaxed long term optimization. The overall better performance of H indicates that SL has a beneficial influence in the final control results since it is able to anticipate fa3rther events and to build the set-point vector of CL considering them. As shown in Figure 3, the long term behavior of the system is implicit incorporated through $WS^y_{m,k'+1}$ (CL set-point), where the change of the user temperature set-point is anticipated. In this case, the big advantage of H is that only the number of the decision variables of the relaxed problem is increased to consider larger prediction horizons, keeping constant the mathematical complexity of the MIP problem of the second layer. DFP has also a

Table 3. Weight of local cost function J_m regarding the global cost function J_g.

Strategy	$J_1^{\%}$	$J_2^{\%}$	$J_3^{\%}$
C	31.39%	31.01%	37.60%
DFP	16.59%	16.59%	66.82%
DCP	39.33%	26.25%	34.42%
H	33.66%	30.91%	35.43%

long term prediction horizon but still global results that are worse than H. To correctly analyze this behavior we have to examine Table 3, where the weight of local cost functions J_m regarding the global cost function J_g is given in percentage for each different strategy. When using the DFP strategy, the value related to the least priority load, represented by $J_3^{\%}$, is considerably higher than for the other devices. This is because the power requirements of the loads that have bigger priority are always the first to be fulfilled and no energy is left for the loads with smaller priority since the available global power is restrained. This unbalanced reparation of the energy strongly increases the dissatisfaction criteria of the least priority load, leading to poor global performances. Table 3 also shows that H is able to share the available power in a most equitable way and to find a energy partition that is as fair as possible with all loads considering the objective of the original global optimization problem. At the end, the cascade priority procedure implemented in H approximates the local solution of CL to the results found with the Centralized strategy.

By reason of simplicity, the size of the system analyzed in this paper is limited, but the same conclusions can be obtained when comparing the approaches for larger study cases.

4.3 Computation time

The results of Table 4 summarizes the average T_g^{mean}, the minimum T_g^{min}, the maximum T_g^{max}, and the standard deviation T_g^{std} of the computation time in seconds needed to find the solution for the 10 different simulations defined in the previous sub-section. The MOSEK MIP optimizer employs a relaxed feasibility and optimality criterion to determine when a sufficient good solution is located. In our case, all termination parameters are set to their default values. As we can remark, the computation time required

Table 4. Average value, minimum value, maximum value, and standard deviation of the computation time in seconds.

Strategy	T_g^{mean}	T_g^{min}	T_g^{max}	T_g^{std}
C	3006.82	0.47	3681.36	1116.17
DFP	0.60	0.10	1.45	0.42
DCP	0.23	0.19	0.28	0.03
H	0.27	0.21	0.46	0.07

to find a satisfactory solution when using the Centralized strategy is much higher than for the other approaches. Since all sub-systems are considered in the same long term optimization, the search space of the problem is increased and thus the mathematical complexity of the solution. This behavior is especially prohibitive for problems involving a large number of sub-systems or longer prediction horizons. In Table 5, for example, the values of J_g^{mean} and T_g^{mean} for Δh = $24h$ are presented. We can observe that the strategies C and DFP are intractable. Contrary

Table 5. Global cost function value and computation time in seconds for the 24-hours simulation.

Strategy	J_g^{mean}	T_g^{mean}
C	intractable	intractable
DFP	intractable	intractable
DCP	259.13	2.49
H	203.62	2.67

to MIP problems that are in many practical situations non-deterministic polynomial-time hard (NP-hard), LP problems can be solved efficiently in the worst case with most of commercial solvers. To exploit this fact, H carries out the global long prediction horizon optimization in the relaxed upper layer problem and considers the discrete constraints only in the local problems of the second layer, where thanks to the small prediction horizon and the decentralized architecture the number of decision variables is reduced. In this way, H offers a balanced trade-off between computation overhead and sub-optimality.

5. CONCLUSIONS AND FUTURE WORK

This paper proposed a two-time scale controller that tackles the BEMS problem considering ON/OFF loads and power global constraints. The simulations show that the hierarchical structure is able to satisfactorily approximate the solution to the Centralized strategy results in a reasonable computation time.

Several future works will be performed to validate the robustness of the control strategy under more realistic conditions and to delineate its operating limits. The priority coordination procedure will also be improved in order to dynamically update the priority queue considering the current state of the system.

REFERENCES

Abreu, A., Bourdais, R., and Guéguen, H. (2017). Inter-layer interactions in hierarchical mpc for building energy management systems. In *IFAC Wolrd Congress*, 12533–12538.

Beaudin, M., Zareipour, H., Bejestani, A.K., and Schellenberg, A. (2014). Residential energy management using a two-horizon algorithm. *IEEE Transactions on Smart Grid*, 5(4), 1712–1723.

Haider, H.T., See, O.H., and Elmenreich, W. (2016). A review of residential demand response of smart grid. *Renewable and Sustainable Energy Reviews*, 59, 166–178.

Lefort, A., Bourdais, R., Ansanay-Alex, G., and Guéguen, H. (2013). Hierarchical control method applied to energy management of a residential house. *Energy and Buildings*, 64, 53 – 61.

Liu, X., Ivanescu, L., Kang, R., and Maier, M. (2012). Real-time household load priority scheduling algorithm based on prediction of renewable source availability. *IEEE Transactions on Consumer Electronics*, 58(2).

Lofberg, J. (2004). Yalmip: A toolbox for modeling and optimization in matlab. In *Computer Aided Control Systems Design, 2004 IEEE International Symposium on*, 284–289. IEEE.

Luo, R., Bourdais, R., van den Boom, T.J., and De Schutter, B. (2017). Multi-agent model predictive control based on resource allocation coordination for a class of hybrid systems with limited information sharing. *Engineering Applications of Artificial Intelligence*, 123–133.

Mayer, B., Killian, M., and Kozek, M. (2017). Hierarchical model predictive control for sustainable building automation. *Sustainability*, 9(2), 264.

Pipattanasomporn, M., Kuzlu, M., and Rahman, S. (2012). An algorithm for intelligent home energy management and demand response analysis. *IEEE Transactions on Smart Grid*, 3(4), 2166–2173.

Schirrer, A., Brandstetter, M., Leobner, I., Hauer, S., and Kozek, M. (2016). Nonlinear model predictive control for a heating and cooling system of a low-energy office building. *Energy and Buildings*, 125, 86–98.

Shaikh, P.H., Nor, N.B.M., Nallagownden, P., Elamvazuthi, I., and Ibrahim, T. (2014). A review on optimized control systems for building energy and comfort management of smart sustainable buildings. *Renewable and Sustainable Energy Reviews*, 34, 409 – 429.

Thieblemont, H., Haghighat, F., Ooka, R., and Moreau, A. (2017). Predictive control strategies based on weather forecast in buildings with energy storage system: A review of the state-of-the art. *Energy and Buildings*, 153, 485–500.

Tsui, K.M. and Chan, S.C. (2012). Demand response optimization for smart home scheduling under real-time pricing. *IEEE Transactions on Smart Grid*, 1812–1821.

Yu, Z., Jia, L., Murphy-Hoye, M.C., Pratt, A., and Tong, L. (2013). Modeling and stochastic control for home energy management. *IEEE Transactions on Smart Grid*, 4(4), 2244–2255.