Termination of lambda-calculus linearization methods

Sandra Alves
DCC-FCUP & CRACS, University of Porto

sandra@dcc.fc.up.pt

The notion of linearisation of the lambda-calculus has been explored in different settings: using information on types, given by a particular intersection type system; or using the notion of computation as paths, deriving from Lévy's definition of labelled reduction.

Linearisation as a process of transforming/simulating non-linear functions into/using "equivalent" linear functions, was first introduced by Assaf Kfoury [4]. In his paper Kfoury embedded the λ-calculus into a new calculus, denoted $\Lambda^\wedge$, with a new notion of *"linear"* reduction, denoted $\beta^\wedge$. Kfoury defined a notion of *contraction* of expanded terms in the new calculus into standard λ-terms, and a notion of lifting of β-reductions into $\beta^\wedge$-reductions. Well-formed terms of the new calculus are those for which there is a contracted term in the λ-calculus. Kfoury conjectured that for strongly normalisable lambda-terms there are corresponding well-formed expanded terms.

A slightly different question that was posed in [1], was the possibility to simulate the λ-calculus, by a proper "linear" subset of the calculus. In this case the subset considered was called the *weak linear λ-calculus*. Linearisation from standard λ-terms into weak linear terms was defined by computing legal paths [2] on the initial term, identifying virtual redexes. Similarly to Kfoury's, a conjecture was also stated relating terms for which the termination process terminates, and terms in the strongly normalising λ-calculus.

A linearisation procedure, using information given by intersection types, was defined by Florido and Damas [3], by means of a notion of expansion of terms in the λ-calculus into linear terms. One of the implications of the expansion relation is that, any term typable using intersection types, can be expanded to a term typable using simple types. The relation between linearisation and intersection types was previously established in an indirect way by Kfoury [4] through the $\Lambda^\wedge$ calculus.

In this talk we will explore these previous works, discuss how they are related and present some open problems regarding the termination of linearisation methods.

References

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