The Symmetry Rule for Quantified Boolean Formulas*

Manuel Kauers[1] and Martina Seidl[2]

[1] Institute for Algebra, JKU Linz, Austria
manuel.kauers@jku.at
[2] Institute for Formal Models and Verification, JKU Linz, Austria
martina.seidl@jku.at

One of the oldest and best understood proof system for refuting quantified Boolean formulas of the form $P.\phi$ (P is the prefix, ϕ is a propositional formula in clausal form) is the proof system Q-Res introduced by Kleine Büning et al. [1]. Compared to other proof systems, Q-Res is rather weak. Exponential separations are, for example, obtained by using the inability of Q-Res to give efficient proofs for the KBKF family [1] and the Q-parity family [2].

We suggest to extend Q-Res with an additional rule called *symmetry rule*. This rule has been added to the propositional resoution calculus before [3, 4] resulting in a proof system that is exponentially stronger than plain resolution. Before we introduce the symmetry rule, let us quickly recall the notion of symmetries for QBFs (for the details see [5]. A bijective map σ from the set $\{x_1, \dots, x_n, \bar{x}_1, \dots, \bar{x}_n\}$ of literals to itself is called admissible for a prefix $P = Q_1 x_1 \dots Q_n x_n$ if $\overline{\sigma(x)} \leftrightarrow \sigma(\bar{x})$ for all $x \in \{x_1, \dots, x_n\}$ and for all $i, j \in \{1, \dots, n\}$, we have $\sigma(x_i) \in \{x_j, \bar{x}_j\}$ only if x_i and x_j belong to the same quantifier block. An admissible function σ is called a symmetry for a QBF $P.\phi$ if applying σ to all literals in ϕ maps ϕ to itself. Now we define the symmetry rule as follows:

S From an already derived clause C and a symmetry σ of $P.\phi$, the clause $\sigma(C)$ can be derived.

The proof system that is obtained by adding S to Q-Res is called Q-Res+S. Soundness of Q-Res+S is shown by induction over the number of the applications of S. The power of the symmetry rule becomes obvious when applying Q-Res+S on KBKF and Q-parity as well as on extensions of these. All of these families are easy for Q-Res+S. We immidately get separations to powerful proof systems like other extensions of Q-Res or expansion-based proof systems like IR-calc [2]. As in propositional logic, S is very sensitive to the formula structure. By the introduction of universal variables that are pure into certain clauses, the application of S can be easily prohibited and only the rules of Q-Res can be used. Hence, Q-Res+S is incomparable to many other proof systems.

All details of our work on the symmetry rule are described in [6] (currently under review).

* Parts of this work were supported by the Austrian Science Fund (FWF) under grant numbers NFN S11408-N23 (RiSE), Y464-N18, and SFB F5004.

References

1. Kleine Büning, H., Karpinski, M., Flögel, A.: Resolution for quantified boolean formulas. Inf. Comput. **117**(1) (1995) 12–18
2. Beyersdorff, O., Chew, L., Janota, M.: Proof complexity of resolution-based QBF calculi. In: STACS. Volume 30 of LIPIcs., Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik (2015) 76–89
3. Krishnamurthy, B.: Short proofs for tricky formulas. Acta Inf. **22**(3) (1985) 253–275
4. Urquhart, A.: The symmetry rule in propositional logic. Discrete Applied Mathematics **96-97** (1999) 177–193
5. Kauers, M., Seidl, M.: Symmetries of Quantified Boolean Formulas. CoRR **abs/1802.03993 (preprint; accepted for SAT'18)** (2018)
6. Kauers, M., Seidl, M.: Short proofs for some symmetric quantified boolean formulas. arXiv preprint arXiv:1804.01437 (2018)