Bounded induction without parameters

Emil Jeřábek

Institute of Mathematics of the Academy of Sciences
Žitná 25, 115 67 Praha 1, Czech Republic, email: jerabek@math.cas.cz

In the area of strong fragments of Peano arithmetic, it proved fruitful to study not just the usual induction fragments $I\Sigma_i$, but also fragments axiomatized by *parameter-free* induction schemata

$$(\varphi\text{-}IND^-) \qquad \varphi(0) \land \forall x\,(\varphi(x) \to \varphi(x+1)) \to \forall x\,\varphi(x)$$

where φ has no free variable other than x, and theories axiomatized using the closely related induction *inference rules*

$$(\varphi\text{-}IND^R) \qquad \frac{\varphi(0,\vec{y}) \quad \varphi(x,\vec{y}) \to \varphi(x+1,\vec{y})}{\varphi(x,\vec{y})}.$$

See e.g. [6, 1, 2]; in particular, the last two papers detail the connection of induction rules and parameter-free schemata to reflection principles.

Induction rules and parameter-free induction schemata received much less attention in bounded arithmetic literature. Kaye [5] discussed the parameter-free fragments IE_i^- of $I\Delta_0$. In the framework of Buss's theories, parameter-free fragments were studied in passing by Bloch [3], but the first systematic investigation of them was done by Cordón-Franco, Fernández-Margarit and Lara-Martín [4], who proved, in particular, conservation results for Σ_i^b-induction rules and parameter-free schemata.

This left unanswered many basic questions about the parameter-free fragments. Most importantly, nothing has been published so far about Π_i^b-induction rules and parameter-free schemata, despite that they could be expected to behave rather differently from Σ_i^b rules by analogy with the case of strong fragments.

In this talk, we will have a closer look at some aspects of parameter-free and inference rule versions of the theories T_2^i and S_2^i: that is, Σ_i^b-IND^-, Π_i^b-IND^-, Σ_i^b-IND^R, Π_i^b-IND^R, and the corresponding variants of *PIND*. We are particularly interested in reductions (implications) between the fragments, conservation results, results on the number of instances (nesting) of rules, and connections to propositional reflection principles.

We will present a new witnessing theorem for (unbounded) $\forall\exists\forall\Pi_i^b$-consequences and $\forall\exists\forall\Pi_{i+1}^b$-consequences of the theories T_2^i and S_2^i, which is at the heart of some of our conservation results.

References

1. Beklemishev, L.D.: Induction rules, reflection principles, and provably recursive functions. Annals of Pure and Applied Logic **85**(3), 193–242 (1997)

2. Beklemishev, L.D.: Parameter free induction and provably total computable functions. Theoretical Computer Science **224**, 13–33 (1999)
3. Bloch, S.A.: Divide and conquer in parallel complexity and proof theory. Ph.D. thesis, University of California, San Diego (1992)
4. Cordón-Franco, A., Fernández-Margarit, A., Lara-Martín, F.F.: Existentially closed models and conservation results in bounded arithmetic. Journal of Logic and Computation **19**(1), 123–143 (2009)
5. Kaye, R.: Diophantine induction. Annals of Pure and Applied Logic **46**(1), 1–40 (1990)
6. Kaye, R., Paris, J., Dimitracopoulos, C.: On parameter free induction schemas. Journal of Symbolic Logic **53**(4), 1082–1097 (1988)