Sequentialising nested systems

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Abstract. In this work, we investigate the proof theoretic connections between sequent and nested proof calculi. Specifically, we identify general conditions under which a nested calculus can be transformed into a sequent calculus by restructuring the nested sequent derivation (proof) and shedding extraneous information to obtain a derivation of the same formula in the sequent calculus. These results are formulated generally so that they apply to calculi for intuitionistic, normal and non-normal modal logics.

1 Introduction

Contemporary proof theory can be traced to Gentzen's seminal work [4] where analytic proof calculi for classical and intuitionistic logic were presented. Proof calculi consist of formal rules of inference which describe the logic under consideration; in an analytic proof calculus, every formula that occurs in a proof generated by the calculus is a subformula of the end formula that is proved. Analyticity is crucial because it induces a structure on the proofs (in terms of the end formula). This proof structure can be exploited to formalise reasoning, investigate metalogical properties of the logic e.g. decidability, complexity and interpolation, and develop automated deduction procedures.

The wide applicability of logical methods and their use in new subject areas has resulted in an explosion of new logics different from classical logic; their usefulness depends on the availability of an analytic proof calculus. The sequent calculus is the simplest and best-known formalism for constructing analytic proof calculi. Unfortunately, there are many more natural non-classical logics—for example, most extensions of intuitionistic and modal logic—for which the sequent calculus formalism is unable to provide an analytic calculus (the precise reasons for this inability are still not well understood). In response, many more new formalisms have been proposed, such as the *hypersequent* [8,1], *labelled sequent* [3,7], *nested sequent* [5,2] and *linear nested sequent* [6] formalisms. This work is primarily concerned with the nested sequent formalism which is obtained by replacing the sequent in the sequent calculus with a tree of sequents.

While the trend has been towards developing formalisms with greater sophistication in order to provide non-classical logics with analytic calculi, in this work we look in the reverse direction by investigating which aspects of this sophistication are extraneous. More specifically, we identify general conditions under which a nested calculus can be transformed into a sequent calculus by restructuring the nested sequent derivation (proof) and shedding extraneous information to obtain a derivation of the same formula in the sequent calculus. These results are formulated generally so that they apply to nested sequent calculi for intuitionistic, normal and non-normal modal logics.

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