

# Some incoherent musings on deep inference and combinatorial proofs

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In this talk I shall make every effort to be provocative and controversial, in an attempt to fuel introspection at the 20 year mark of deep inference. The talk has three parts:

(a) Is deep inference “incoherent categorical logic”? Here “incoherent” refers to the omission of proof equality, covered by the coherence laws of categorical logic [1]. Is deep inference in danger of isolating itself as a community by not preserving the original language of categorical logic? Hilbert’s 24th problem is on proof equality [2,3]; does deep inference short-change itself by not retaining proof equality as a first-class citizen, as it was in categorical logic?

(b) Linear distributivity (originally known as weak distributivity) [4] is core to the categorical approach to linear logic, hence also deep inference, where it has become known as *switch*. Classical logic follows with contraction and weakening. Combinatorial proofs [5] are an irredundant abstraction [3] of classical syntactic proofs, yet there is no natural interpretation of switch on combinatorial proofs, while there is a natural interpretation of the sequent calculus conjunction rule. Could there be a deeper computational meaning to this, which reflects back on the standard switch-based formulations of deep inference for linear and classical logic?

(c) Extensions of combinatorial proofs. In part (b) I will have shown how easily one may translate a classical propositional syntactic proof into a combinatorial proof (literally a 30-second definition - I promise!). I will then talk about two extensions, where the translation from syntactic proofs is also very easy to explain to a lay audience: (a) intuitionistic combinatorial proofs (joint work with Heijltjes and Strassburger), and (b) classical first-order combinatorial proofs [6].

*References and background reading:*

[1] Dominic J. D. Hughes. Deep inference proof theory equals categorical proof theory minus coherence. Technical report, October 2004. <http://boole.stanford.edu/~dominic/papers/di/di.pdf>

[2] Rüdiger Thiele. Hilbert’s Twenty-fourth Problem. American Mathematical Monthly, January 2003. [https://www.maa.org/sites/default/files/pdf/upload\\_library/22/Ford/Thiele1-24.pdf](https://www.maa.org/sites/default/files/pdf/upload_library/22/Ford/Thiele1-24.pdf)

[3] Dominic J. D. Hughes. Towards Hilbert’s 24th problem: combinatorial proof invariants. Proc. WoLLIC’06, ENTCS 165, 2006. <http://boole.stanford.edu/~dominic/papers/invar/invar.pdf>

[4] Robin Cockett and Robert Seely. Weakly Distributive Categories. Journal of Pure and Applied Algebra 114:2 1997. <http://www.math.mcgill.ca/rags/linear/wdc.ps.gz>

[5] Dominic J. D. Hughes. Proofs without syntax. Annals of Mathematics 164 2006. <http://annals.math.princeton.edu/wp-content/uploads/annals-v164-n3-p09.pdf>  
<http://arxiv.org/pdf/math/0408282.pdf>

[6] Dominic J. D. Hughes. First-order Proofs Without Syntax. Slides for the Berkeley Logic Colloquium, 17 October 2014. <http://boole.stanford.edu/~dominic/papers/fopws-blc-2014>

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\*Visiting scholar. Many thanks to my host, Wes Holliday.