Modelling SAT

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Abstract. Satisfiability (SAT) is the canonical NP-complete problem [3, 5]. But how hard is it exactly, and on which instances? There are several existing approaches that aim to analyze and understand the complexity of SAT:

1. Proof complexity [2]: Here the main goal is to show good lower bounds on the sizes of proofs for unsatisfiability of certain "hard" formulas in various proof systems. The importance of this approach owes partly to the fact that lower bounds on proof size can be translated into lower bounds on running time for SAT solvers of interest.
2. Exact algorithms [4]: Here the goal is to get better upper bounds on worst-case running time for SAT algorithms. Assuming $\mathsf{NP} \neq \mathsf{P}$, these improved upper bounds will not be polynomial, but they could still improve substantially over the naive brute force search algorithm.
3. Random SAT [1]: Here the goal is to understand the hardness of SAT on random instances, where clauses are picked independently and uniformly at random. The methods of statistical physics turn out to be helpful - physical insights about phase transitions and the structure of solution spaces can be used to quantify the performance of a large class of algorithms.

I broadly discuss these approaches and the relationships between them. I suggest that their complementary perspectives could be useful in developing new models and answering questions that do not seem to be answerable by any individual approach. Indeed SAT serves partly as an excuse to investigate a larger issue: what are good algorithmic models, and what questions should we be asking about them?

Keywords: Satisfiability, modelling, proof complexity, exact algorithms, phase transitions

References

1. Achlioptas, D.: Random Satisfiability. Handbook of Satisfiability, 245–270 (2009)
2. Beame, P., Pitassi, T.: Propositional Proof Complexity: Past, Present, and Future. Current Trends in Theoretical Computer Science, 42–70 (2001)
3. Cook, S.: The Complexity of Theorem-Proving Procedures. Proceedings of 3rd Annual ACM Symposium on Theory of Computing (STOC), 151–158 (1971)
4. Dantsin, E., Hirsch, E.: Worst-Case Upper Bounds. Handbook of Satisfiability, 403–424 (2009)
5. Levin, L.: Universal Sequential Search Problems. Problemy Peredachi Informatsii, 9(3), 115–116 (1973)