Constraint Problem for Weak Subgame Perfect Equilibria with ω-regular Boolean Objectives

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Two-player zero-sum turn-based games played on graphs are commonly used to model interactions between a computer system and its environment: the system wants to achieve a goal - to satisfy a certain property - and the environment acts in an antagonistic way [6, 4]. When modelling interacting computer systems this kind of model is too restrictive, and a setting with more than two agents whose objectives are not necessarily antagonistic is more realistic. These games are called *multiplayer non zero-sum turn-based games* and agents are modelled by *players* [2]. In such a game, each player chooses a *strategy*: it is the way he plays given some information about the game and past actions of all players. Given a fixed strategy per player, this set of strategies is called a *strategy profile*, written $\overline{\sigma}$. Following a strategy profile $\overline{\sigma}$ from an *initial vertex* v_0 results in an infinite sequence of vertices of the graph game called a *play*. This play is denoted by $\langle\overline{\sigma}\rangle_{v_0}$. Given a play, a gain for each player can be associated with it. We use the notation $\mathrm{Gain}(\langle\overline{\sigma}\rangle_{v_0})$ to express the gain profile of players associated with the play $\langle\overline{\sigma}\rangle_{v_0}$, *i.e.,* it is a tuple such that each component corresponds to the gain of a player. In this paper we focus on *ω-regular Boolean objectives*: either a player satisfies his objective (gain of 1) or he does not (gain of 0).

In this multiplayer game setting, one classical solution concept is the concept of *Nash equilibrium* (NE). Roughly, a Nash equilibrium is a strategy profile such that no player has incentive to deviate unilaterally from his strategy to obtain a better gain. In other words no player has a *profitable deviation.* In the context of games played on graphs, a more relevant solution concept is the one of *subgame perfect equilibrium* (SPE) because it takes into account the sequential structure of the game. *Weak Nash equilibria* (weak NE) are a recent variant of NE introduced in [1, 3], in which motivations for considering weak NE can be found. The difference between weak NE and NE is that players can only use profitable deviations with a finite number of deviations from the initial strategy profile. In this paper, we are interested in the concept of *weak subgame perfect equilibrium* (weak SPE). Weak NE are to weak SPE what NE are to SPE. We illustrate the difference between NE and weak NE in Figure 1. In this game, player P_1 (resp. P_2) owns round vertices (resp. square vertices) and wants to reach v_1 (resp. v_3) infinitely often. The strategy profile $\overline{\sigma}$ is depicted by dashed arrows and the play resulting in following it from v_0 is $\langle\overline{\sigma}\rangle_{v_0} = v_0 v_1 v_2 v_3^{\omega}$. Notice that along this play only player P_2 satisfies his objective and P_1 has an incentive to deviate from his strategy. In order to satisfy his objective, P_1 can repeatedly go to v_1 each time he has a choice to make, and this is the only profitable deviation. So, the strategy profile depicted by dashed arrows is not an NE. Nevertheless, it is a weak NE because always switching to v_1 in place of v_3

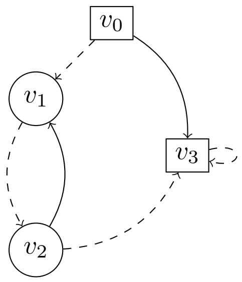

Figure 1: Example of a weak NE which is not an NE.

occurs an infinite number of times from the initial strategy profile.

The *constraint problem* for equilibria is a natural problem already studied in [5]. It asks to determine whether there exists an equilibrium such that the gain of the players satisfies some constraints, *i.e.,* given a game with n players, an initial vertex v_0 and two Boolean thresholds $\overline{x}, \overline{y} \in \{0,1\}^n$, the constraint problem for weak SPE asks whether there exists a weak SPE $\overline{\sigma}$ such that $\overline{x} \leq \text{Gain}(\langle\overline{\sigma}\rangle_{v_0}) \leq \overline{y}$.

The goal of our ongoing research is to characterise the complexity of the constraint problem for the class of games with ω-regular Boolean objectives. We are interested about classical objectives: Explicit Muller, Muller, Büchi, co-Büchi, Parity, Streett, Rabin, Reachability and Safety. In our setting, all players have the same type of objective; for example, as illustrated in Figure 1, all players have a Büchi objective. In this sense, the games we study are games with multiple objectives.

Up to now, the results we have obtained for classical ω-regular Boolean objectives are shown in Table 1. Exact complexity for Büchi objectives remains open. We only obtained a non-deterministic algorithm in polynomial time. Moreover, as SPE and weak SPE are equivalent for Reachability (Boolean) objectives, it proves that the constraint problem is PSPACE-complete for SPE too. This is interesting since, to our knowledge, the exact complexity for the constraint problem in the case of Reachability Boolean SPE was unknown.

Table 1: Complexity of the constraint problem for ω-regular Boolean objectives.

	exp. Muller	Muller	co-Büchi	Parity	Streett	Rabin	Reach	Safety
P	×							
NP-c		×	×	×	×	×		
PSPACE-c							×	×

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