A Resolution-Based Calculus for Preferential Logics*

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Abstract. The vast majority of modal theorem provers implement modal tableau, or backwards proof search in (cut-free) sequent calculi. The design of suitable calculi is highly non-trivial, and employs nested sequents, labelled sequents and/or specifically designated transitional formulae. Theorem provers for first-order logic, on the other hand, are by and large based on resolution. In this paper, we present a resolution system for preference-based modal logics, specifically Burgess' system S. Our main technical results are soundness and completeness. Conceptually, we argue that resolution-based systems are not more difficult to design than cut-free sequent calculi but their purely syntactic nature makes them much better suited for implementation in automated reasoning systems.

1 Introduction

Theorem-provers for First-Order logic, such as E [20], Vampire [17] and SPASS [22] are typically based on resolution, often augmented with elements of the superposition calculus [1] to deal with equality. This is in sharp contrast with Modal (or Description Logic) reasoners which are typically based on variants of analytic tableau. Examples are the FACT++ reasoner [21], LoTREC [7], LeanTAP [2] and Racer [11]. The situation is similar for non-normal modal logics, such as Alternating Temporal Logic [5] and various forms of conditional logics [14,15] as well as various logics that can be subsumed under co-algebraic semantics [10]. Modal theorem provers based on resolution, on the other hand, are thin on the ground, but compare favourably with Tableau-based approaches in terms of efficiency [12,13].

As part of an ongoing investigation into resolution theorem-proving for modal logics, this paper presents a resolution system for Burgess' system S [3], a conditional logic that extends classical propositional logic with a binary modal connective, written $\Rightarrow$, and read as 'if ... then typically ...'. The binary connective $\Rightarrow$ is interpreted over models having a set W of possible worlds, and a preorder relation $\leq_w$ at each world $w \in W$. The preorder relation can be interpreted as local plausibility relation, where $w' \leq_w w''$ is interpreted as w' being as plausible as w'' (from the perspective of w). In finite models, the modal formula $\phi \Rightarrow \psi$ can then be interpreted at w by stipulating that

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every $\leq_w$-minimal world that satisfies ϕ must also satisfy ψ. The dual of $\Rightarrow$ is denoted by $\not\Leftarrow$, that is, $\varphi \not\Leftarrow \psi$ is defined as $\neg\varphi \Rightarrow \neg\psi$. The interpretation of $\varphi \not\Leftarrow \psi$ is, thus, that there exists a minimal $\neg\varphi$-world which satisfies ψ.

The ensuing logic is part of a family of conditional logics [4] for which sequent, or tableau calculi are notoriously hard to construct, and often require additional syntactic structure. Various conditional logics require nested sequents [14], labelled sequents [9,8] or special transition formulae [15], together with non-trivial proofs of either semantic completeness or cut elimination. Again, this is in sharp contrast to modal calculi based on resolution, where the only extra machinery needed is a global modality.

Our main technical contribution is the design of a resolution calculus for Burgess' system S, together with proofs of soundness and completeness. As with other resolution-based systems found in the literature (including First-Order resolution calculi), our procedure consists of two phases. In the first phase, an input formula is translated into an equisatisfiable set of clauses. Then a set of inference rules is applied to the clause set. There are two types of rules: one corresponding to the usual modal propagation, as seen in modal tableaux calculi; and a set of resolution-based rules. Although the method presented here is essentially clausal, the formula φ in a modal formula of the form $\varphi \not\Leftarrow \psi$ partially retains the structure of the original problem on the left-hand side of the modal operator. This allows for a simpler set of rules for modal propagation based on the set of axioms for S. Besides the resolution-based rules for dealing with the propositional fragment of the logic, the resolution rules operate on modal formulae and propagate potential inconsistencies between modal formulae to the propositional level.

Conceptually, we argue that resolution-based systems are not more difficult to design than cut-free sequent calculi but their purely syntactic nature makes them much better suited for implementation in automated reasoning systems.

The paper is organised as follows. In the next section, the language of S is given, following the presentation in [6]. The resolution-based calculus for S, named $\mathsf{RES_S}$, is detailed in Section 3: we present the transformation rules for translating a formula into the normal form and the inference rules of $\mathsf{RES_S}$, together with a non-trivial example involving nested conditional formulae. In Section 4, we show that $\mathsf{RES_S}$ is sound, complete, and terminating. We summarise and discuss our results in Section 5.

2 Language

In this section we introduce the language of S, following closely the presentation in [6]. Let $\mathsf{P} = \{p, q, r, \ldots, p', q', r', \ldots\}$ be a denumerable set of propositional symbols. Formulae are built from P, the usual classical connectives for negation ($\neg$) and conjunction ($\wedge$), and the conditional implication ($\Rightarrow$). The set of well-formed formulae of S, denoted by $\mathsf{WFF_S}$, is inductively defined as follows:

- for all $p \in \mathsf{P}$, $p \in \mathsf{WFF_S}$;
- if φ_i, $0 \leq i \leq n$, $n \in \mathbb{N}$, are in $\mathsf{WFF_S}$, then so are $\neg\varphi_1$, $(\varphi_1 \wedge \ldots \wedge \varphi_n)$, and $\varphi_1 \Rightarrow \varphi_2$.

The empty conjunction is denoted by **true** (*verum*). Let φ_i, $0 \leq i \leq n$, $n \in \mathbb{N}$, be formulae in $\mathsf{WFF_S}$. The following connectives are introduced as abbreviations: **false** $= \neg$**true** (*falsum*), $(\varphi_1 \vee \ldots \vee \varphi_n) = \neg(\neg\varphi_1 \wedge \ldots \wedge \neg\varphi_n)$ (disjunction), $(\varphi_1 \rightarrow \varphi_2) = (\neg\varphi_1 \vee \varphi_2)$

(implication), and $(\varphi_1 \leftrightarrow \varphi_2) = (\varphi_1 \to \varphi_2) \wedge (\varphi_2 \to \varphi_1)$ (double implication). We denote the dual of $\Rightarrow$ by $\not\Rightarrow$, that is, $\varphi_1 \not\Rightarrow \varphi_2$ is defined as $\neg(\neg\varphi_1 \Rightarrow \neg\varphi_2)$. Parentheses are omitted if the reading is not ambiguous. We set the precedence order of operators as $\neg < \{\wedge, \vee\} < \to < \leftrightarrow < \{\Rightarrow, \not\Rightarrow\}$, that is $\neg$ binds stronger than $\wedge$ and $\vee$, which bind stronger than $\to$, as usual.

A *literal* is a proposition or its negation. We denote the set of all literals by L. The set of subformulae of a formula is defined in the usual way. As we take the conjunction as an n-ary operator, for a formula φ of the form $\varphi_1 \wedge \ldots \wedge \varphi_n$, any conjunction formed by the subformulae occurring in φ is a subformula of φ. For instance, p, q, r, $p \wedge q$, $p \wedge r$, $q \wedge r$, and $p \wedge q \wedge r$ are all the subformulae of $p \wedge q \wedge r$.

A complete axiomatisation for S is given in [3] and comprises the following axiom schemata (where $\varphi, \psi, \chi \in \mathsf{WFF_S}$):

A0 all propositional tautologies;
A1 $\varphi \Rightarrow \varphi$ (reflexivity);
A2 $((\varphi \Rightarrow \psi) \wedge (\varphi \Rightarrow \chi)) \to (\varphi \Rightarrow (\psi \wedge \chi))$;
A3 $(\varphi \Rightarrow (\psi \wedge \chi)) \to (\varphi \Rightarrow \psi)$ (monotonicity on the right-hand side of $\Rightarrow$);
A4 $((\varphi \Rightarrow \psi) \wedge (\varphi \Rightarrow \chi)) \to ((\varphi \wedge \psi) \Rightarrow \chi)$ (cautious monotonicity);
A5 $((\varphi \Rightarrow \chi) \wedge (\psi \Rightarrow \chi)) \to ((\varphi \vee \psi) \Rightarrow \chi)$ (or);

together with uniform substitution and the following inference rules: *modus ponens* [MP] if $\vdash \varphi$ and $\vdash (\varphi \to \psi)$, then $\vdash \psi$; and *replacement of provable equivalents* [RPE] if $\vdash (\varphi_1 \leftrightarrow \varphi_2)$ and $\vdash \psi$, then $\vdash \psi'$, where ψ' only differs from ψ by replacing some subformulae of ψ of the form φ_1 by φ_2.

The semantics of S is given in terms of Kripke structures with a ternary relation over worlds. Let (W, w_0, π, R) be a Kripke structure where $W \neq \emptyset$ is a set (of *worlds*) with a distinguished world w_0; $\pi : W \longrightarrow (\mathsf{P} \longrightarrow \{\mathit{true}, \mathit{false}\})$ is an evaluation function which maps every world to a truth assignment over P; and R is a ternary relation over W, where $\leq_w = \{(w', w'') \mid (w, w', w'') \in R\}$, for which $\leq_w$ is a preorder (i.e. a reflexive and transitive relation). We say that $\leq_w$ is a *preferential order* over the worlds in W from the point of view of w. We define W_w, for $w \in W$, to be the set $\{w' \mid (w', w'') \in \leq_w$, for some $w''\}$, that is, W_w is the set of worlds considered at least as plausible as some world in W according to the preferential order given by $\leq_w$.

Let $M = (W, w_0, \pi, R)$ be a Kripke structure. Truth of a formula at a world $w \in W$ in M, denoted by $\models$, is defined as follows (where $\varphi_i \in \mathsf{WFF_S}$, for all $0 \leq i \leq n$, $n \in \mathbb{N}$):

- $\langle M, w \rangle \models p$ if, and only if, $\pi(w)(p) = \mathit{true}$, for all $p \in \mathsf{P}$;
- $\langle M, w \rangle \models \neg\varphi_1$ if, and only if, $\langle M, w \rangle \not\models \varphi_1$;
- $\langle M, w \rangle \models (\varphi_1 \wedge \ldots \wedge \varphi_n)$ if, and only if, $\langle M, w \rangle \models \varphi_i$, for all $1 \leq i \leq n$;
- $\langle M, w \rangle \models \varphi_1 \Rightarrow \varphi_2$ if, and only if, for all $w' \in W_w$, if $\langle M, w' \rangle \models \varphi_1$, then there is $w'' \in W$ such that $w'' \leq_w w'$ and $\langle M, w'' \rangle \models \varphi_1 \wedge \varphi_2$; and there is no world $w''' \in W_w$ such that $w''' \leq_w w''$ and $\langle M, w''' \rangle \models \varphi_1 \wedge \neg\varphi_2$.

Satisfiability of a formula is given with respect to the distinguished world w_0 in a structure (W, w_0, π, R). A formula φ is *satisfied in a structure* M if $\langle M, w_0 \rangle \models \varphi$. In this case, we say that $\langle M, w_0 \rangle$ is a model for φ. A formula φ is *satisfiable* if there is a model for φ. A formula φ is *valid* if it is satisfiable in all structures. A set of formulae $\Gamma = \{\gamma_1, \ldots, \gamma_n\}$, $n \in \mathbb{N}$, is satisfiable if, and only if, $\bigwedge_{i=1}^{n} \gamma_i$ is satisfiable.

Some further conditions can be imposed on the class of Kripke structures that characterise the semantics of S without affecting the set of valid formulae. For instance, only finite structures need to be considered, as the finite model property for preferential logics holds [3,6]. For finite structures, the interpretation of $\Rightarrow$ is much simpler: $\varphi \Rightarrow \psi$ is satisfiable at a world w if, and only if, all minimal φ-worlds in W_w satisfy $\varphi \wedge \psi$. Let S_n be the sublanguage of S with bounded nesting of at most n preferential operators. We recall the following lemma:

Lemma 1. *[6, Lemma 3.1] Let φ be a formula of the form $(\psi_0 \not\Rightarrow \psi'_0) \wedge \bigwedge_{i=1}^{k}(\psi_i \Rightarrow \psi'_i)$ where $\psi_i, \psi'_i \in \mathsf{S}_0$, for all i, $0 \leq i \leq k$, $k \in \mathbb{N}$. If φ is satisfiable, then φ is satisfiable in a Kripke structure with at most $k+1$ worlds which are totally ordered by $\leq$.*

The proof of Lemma 1, as given in [6], shows that only total orderings over the set of worlds need to be considered when checking the satisfiability of a formula with only one occurrence of the dual of the conditional implication. For formulae with more occurrences of the dual operator, a disjoint set of total orderings over the set of worlds need to be considered, one for each negated conditional. Still, a structure satisfying such a formula is polynomially bounded in the size of the formula [6, Proposition 3.2]. For the language of S_n, with bounded nesting of at most n preferential operators, $n > 1$, testing the satisfiability of a formula can be restricted to structures which are polynomial in the size of the formula, where the degree of the polynomial is bounded by $2 \times n$ [6, Proposition 3.6]. As given in [6], the satisfiability problem for S is PSPACE-complete.

3 Calculus

In this section, we present the clausal resolution-based calculus RES_S for checking the satisfiability of formulae in the language of S. A *clause* is a disjunction of literals or modal formulae of the form $(\varphi \Rightarrow \varphi')$ or $(\varphi \not\Rightarrow \varphi')$, where φ and φ' have no subformulae whose main operator is $\Rightarrow$ or $\not\Rightarrow$. A *literal clause* is a clause with no occurrences of modal operators, that is, it is a disjunction of literals. A formula is in Conjunctive Normal Form (CNF) if, and only if, it is a conjunction of initial and global clauses, defined as follows:

initial clause: $\bigvee_{a=1}^{n} l_a$
global clause: $\boxed{*}(\bigvee_{b=1}^{m_1} l'_b \vee \bigvee_{c=1}^{m_2}(\varphi_c \Rightarrow \psi_c) \vee \bigvee_{d=1}^{m_3}(\varphi'_d \not\Rightarrow \psi'_d))$

where $n, m_1, m_2, m_3 \in \mathbb{N}$, $l_a, l'_b \in \mathsf{L}$, $\varphi_c, \psi_c, \psi'_d$ are literal clauses, φ'_d is in Negation Normal Form (NNF), no formulae contains nested modal operators, and $\boxed{*}$ is the universal operator. We introduce the universal operator because the translation into the normal form requires that the definition of formulae being renamed is available throughout the whole model. The universal operator is interpreted as usual: if $M = (W, w_0, \pi, R)$ is a Kripke structure and $w \in W$, then $\langle M, w \rangle \models \boxed{*}\,\varphi$ if, and only if, for all $w' \in W$, $\langle M, w' \rangle \models \varphi$. The empty clause is denoted by **false**. The transformation into the normal form uses rewriting and renaming, where the renaming technique is used to replace

$$
\begin{array}{rlrl}
\mathsf{nnf}(l) & = l \text{ for } l \in \mathsf{L} & \mathsf{nnf}(\neg\neg\varphi) & = \mathsf{nnf}(\varphi) \\
\mathsf{nnf}(\bigwedge \varphi_i) & = \bigwedge \mathsf{nnf}(\varphi_i) & \mathsf{nnf}(\neg(\bigwedge \varphi_i)) & = \bigvee \mathsf{nnf}(\neg\varphi_i) \\
\mathsf{nnf}(\bigvee \varphi_i) & = \bigvee \mathsf{nnf}(\varphi_i) & \mathsf{nnf}(\neg(\bigvee \varphi_i)) & = \bigwedge \mathsf{nnf}(\neg\varphi_i) \\
\mathsf{nnf}(\varphi \to \psi) & = \mathsf{nnf}(\neg\varphi) \vee \mathsf{nnf}(\psi) & \mathsf{nnf}(\neg(\varphi \to \psi)) & = \mathsf{nnf}(\varphi) \wedge \mathsf{nnf}(\neg\psi) \\
\mathsf{nnf}(\varphi \Rightarrow \psi) & = \mathsf{nnf}(\varphi) \Rightarrow \mathsf{nnf}(\psi) & \mathsf{nnf}(\neg(\varphi \Rightarrow \psi)) & = \mathsf{nnf}(\neg\varphi \not\Rightarrow \neg\psi) \\
\mathsf{nnf}(\varphi \not\Rightarrow \psi) & = \mathsf{nnf}(\varphi) \not\Rightarrow \mathsf{nnf}(\psi) & \mathsf{nnf}(\neg(\varphi \not\Rightarrow \psi)) & = \mathsf{nnf}(\neg\varphi \Rightarrow \neg\psi)
\end{array}
$$

$$\mathsf{nnf}(\varphi \leftrightarrow \psi) = \mathsf{nnf}(\varphi \to \psi) \wedge \mathsf{nnf}(\psi \to \varphi)$$

$$\mathsf{nnf}(\neg(\varphi \leftrightarrow \psi)) = \mathsf{nnf}(\varphi \wedge \neg\psi) \vee \mathsf{nnf}(\psi \wedge \neg\varphi)$$

Table 1. NFF transformation rules

complex formulae in the scope of the disjunctions (except for the left-hand side of the dual operator) and the nesting of conditional operators by new propositional symbols. Clauses and formulae within the scope of modal operators are required to be in simplified form, that is, $\varphi \vee \varphi$, $\varphi \wedge \varphi$, $\varphi \vee \mathbf{false}$, and $\varphi \wedge \mathbf{true}$ simplify to φ; $\varphi \vee \neg\varphi$, $\varphi \vee \mathbf{true}$, and $\boxast\, \mathbf{true}$ simplify to **true**; and $\varphi \wedge \neg\varphi$, $\varphi \wedge \mathbf{false}$, and $\boxast\, \mathbf{false}$ simplify to **false**.

The transformation of a formula φ in the language of S into CNF is given as follows. We denote the NNF of φ by $\mathsf{nnf}(\varphi)$, which is obtained by applying the function $\mathsf{nnf} : \mathsf{WFF_S} \longrightarrow \mathsf{WFF_S}$ to φ, whose definition is given in Table 1, where l is a literal, $\varphi, \varphi_i, \psi \in \mathsf{WFF_S}$ and $i \in \mathbb{N}$. Let φ be a well-formed formula in NNF. The translation of φ into Conjunctive Normal Form is defined as

$$\mathsf{cnf}(\varphi) = t_0 \wedge \tau(\boxast(t_0 \to \varphi))$$

where t_0 is a new propositional symbol and the transformation function $\tau : \mathsf{WFF_S} \longrightarrow \mathsf{WFF_S}$ is defined as follows (where $\varphi, \varphi_i, \psi, \chi \in \mathsf{WFF_S}$, $i \in \mathbb{N}$, t is a literal, and t' is a new propositional symbol). For the base case, the right-hand side of the implication is a disjunction where each disjunct is a literal, or it is of the form $(\varphi' \Rightarrow \varphi'')$ or $(\psi' \not\Rightarrow \psi'')$, where ψ' is a propositional formula (i.e. with no occurrences of subformulae whose main operator is either $\Rightarrow$ or $\not\Rightarrow$), and the formulae φ', φ'' and ψ'' are literal clauses:

$$\tau(\boxast(t \to \varphi)) = \boxast(\neg t \vee \varphi)$$

If the right-hand side of the implication or the right-hand side of a conditional is a conjunction, then rewriting is applied:

$$\tau(\boxast(t \to \bigwedge \varphi_i)) = \bigwedge \tau(\boxast(t \to \varphi_i))$$
$$\tau(\boxast(t \to (\varphi \Rightarrow \bigwedge \varphi_i))) = \bigwedge \tau(\boxast(t \to (\varphi \Rightarrow \varphi_i)))$$

If any of the disjuncts on the right-hand side of the implication is not a literal, then renaming is applied:

$$\tau(\boxast(t \to \bigvee \varphi_i \vee \psi)) = \tau(\boxast(t \to \bigvee \varphi_i \vee t')) \wedge \tau(\boxast(\mathsf{nnf}(t' \leftrightarrow \psi)))$$

Conjunctions on the left-hand side of conditionals and on the right-hand side of the dual operator are renamed as follows:

$$\tau(\boxast(t \to (\bigwedge \varphi_i \Rightarrow \chi))) = \tau(\boxast(t \to (t' \Rightarrow \chi))) \wedge \tau(\boxast(\mathsf{nnf}(t' \leftrightarrow \bigwedge \varphi_i)))$$
$$\tau(\boxast(t \to (\varphi \not\Rightarrow \bigwedge \varphi_i))) = \tau(\boxast(t \to (\varphi \not\Rightarrow t'))) \wedge \tau(\boxast(\mathsf{nnf}(t' \leftrightarrow \bigwedge \varphi_i)))$$

If any of the disjuncts on the left-hand side of the conditional or on the right-hand side of a (negated) conditional is not a literal clause, that is, if ψ in the following is not a literal, then renaming is also applied.

$$\tau(\boxed{*}(t \to ((\bigvee \varphi_i \vee \psi) \Rightarrow \chi))) = \tau(\boxed{*}(t \to ((\bigvee \varphi_i \vee t') \Rightarrow \chi))) \wedge \tau(\boxed{*}(\mathsf{nnf}(t' \leftrightarrow \psi)))$$
$$\tau(\boxed{*}(t \to (\varphi \Rightarrow (\psi \vee \bigvee \varphi_i)))) = \tau(\boxed{*}(t \to (\varphi \Rightarrow (t' \vee \bigvee \varphi_i)))) \wedge \tau(\boxed{*}(\mathsf{nnf}(t' \leftrightarrow \psi)))$$
$$\tau(\boxed{*}(t \to (\varphi \not\Rightarrow (\psi \vee \bigvee \varphi_i)))) = \tau(\boxed{*}(t \to (\varphi \not\Rightarrow (t' \vee \bigvee \varphi_i)))) \wedge \tau(\boxed{*}(\mathsf{nnf}(t' \leftrightarrow \psi)))$$

If the left hand-side of the negated conditional is not a propositional formula, that is, if ψ is of the form $(\psi' \Rightarrow \psi'')$ or $(\psi' \not\Rightarrow \psi'')$, then renaming is also applied. Let $\varphi[\psi \mapsto t']$ denote the result of replacing some occurrences of the subformula ψ in φ by t':

$$\tau(\boxed{*}(t \to (\varphi \not\Rightarrow \chi))) = \tau(\boxed{*}(t \to (\varphi[\psi \mapsto t'] \not\Rightarrow \chi))) \wedge \tau(\boxed{*}(\mathsf{nnf}(t' \leftrightarrow \psi)))$$

Although the resolution-based method for S is essentially clausal, note that formulae on the left-hand side of the modal operator $\not\Rightarrow$ are not required to be literal clauses. This helps preserving more of the structure of the original formula and, as so, to identifying the cases where the axioms **A4** and **A5** should be propagated. We also note that, for a formula φ being renamed by a propositional symbol t, if φ occurs only with positive (resp. negative) polarity, then only the implication $t \to \varphi$ (resp. $(\varphi \to t)$) is needed [16]. As formulae on the left-hand side of conditionals occur with both polarities, in order to simplify our presentation, we have chosen to introduce both sides of the definition of φ, i.e. $(t \to \varphi)$ and $(\varphi \to t)$, into the clause set.

As checking the satisfiability of a conjunction $\bigwedge_{i=1}^{n} \varphi_i$, $n \in \mathbb{N}$, is equivalent to checking the satisfiability of the set $\{\varphi_1, \dots, \varphi_n\}$, we refer to a formula into CNF as a *set of clauses*. Given a set of clauses in CNF, the resolution procedure is applied until a contradiction, in the form of **false**, is found or no new clauses can be derived. The inference rules can be divided into two sets: a set of rules for propagation of formulae whose main operator are either the conditional or its dual; and a set of resolution-based rules.

The inference rules used for propagation given in Table 2 are closely related to the axioms of S. The inference rule [L-OR-2] is related to cautious monotonicity (axiom **A4**). The inference rule [L-AND-2] is related to the disjunction property on the left-hand side of conditional (axiom **A5**). The inference rules [REF-1] and [REF-2] correspond to reflexivity, that is, the axiom **A1**. Finally, the [SIMP-1] corresponds to simplification, as $\varphi \not\Rightarrow$ **false** is unsatisfiable.

The resolution-based inference rules are also given in Table 2, where [I-RES-1] and [I-RES-2], the resolution rules related to *initial* clauses, and [RES] are syntactical variations of the classical binary resolution rule given in [18]. The remaining resolution-based inference rules are justified by the axioms **A2** and **A3**. The resolution-based rule [R-RES-$\Rightarrow$-1] says that when the left-hand side of the conditionals in the premises are equivalent, then the standard binary resolution rule can be applied to the right-hand side of those conditionals. Note that in the case of [R-RES-$\not\Rightarrow$-1], which is similar, the negated conditional in the premises is of the form $(\varphi' \not\Rightarrow \chi')$ and, from the definition of the dual, we have that this conditional is equivalent to $\neg(\neg\varphi' \Rightarrow \neg\chi')$. The disjunct $\neg(\neg\varphi \leftrightarrow \varphi')$ in the conclusion then states that either φ' is not equivalent to the negation of φ (from the other premise) or resolution can be applied to the right-hand side of those conditionals. The inference rule [L-RES-$\not\Rightarrow$] says that if a formula φ cannot be satisfied in any ordering, as given by the premise $\varphi \Rightarrow$ **false**, then any negated conditional whose left-hand side is equivalent to $\neg\varphi$ cannot be satisfied either. The inference rules [R-RES-$\Rightarrow$-2] and [R-RES-$\not\Rightarrow$-2] apply resolution to a literal occurring in a global literal clause with its complement occurring on the right-hand side of conditionals.

[L-AND-2]
$$\frac{\boxed{*}\,(D \vee ((\varphi \wedge \psi) \not\Rightarrow \chi))}{\boxed{*}\,(D \vee (\varphi \not\Rightarrow \chi) \vee (\psi \not\Rightarrow \chi))}$$

[L-OR-2]
$$\frac{\boxed{*}\,(D \vee ((\varphi \vee \psi) \not\Rightarrow \chi))}{\boxed{*}\,(D \vee (\varphi \not\Rightarrow \psi) \vee (\varphi \not\Rightarrow \chi))}$$

[REF-1]
$$\frac{\boxed{*}\,(D \vee (\varphi \Rightarrow \psi))}{\boxed{*}\,(\varphi \Rightarrow \varphi)}$$

[REF-2]
$$\frac{\boxed{*}\,(D \vee (\varphi \not\Rightarrow \psi))}{\boxed{*}\,(\neg\varphi \Rightarrow \neg\varphi)}$$

[SIMP-1]
$$\frac{\boxed{*}\,(D \vee (\varphi \not\Rightarrow \mathbf{false}))}{\boxed{*}\,(D)}$$

[I-RES-1]
$$\frac{\begin{array}{c} D \vee l \\ D' \vee \neg l \end{array}}{D \vee D'}$$

[I-RES-2]
$$\frac{\begin{array}{c} D \vee l \\ \boxed{*}\,(D' \vee \neg l) \end{array}}{D \vee D'}$$

[RES]
$$\frac{\begin{array}{c} \boxed{*}\,(D \vee l) \\ \boxed{*}\,(D' \vee \neg l) \end{array}}{\boxed{*}\,(D \vee D')}$$

where D' is a literal clause

[R-RES-⇒-1]
$$\frac{\begin{array}{c} \boxed{*}\,(D \vee (\varphi \Rightarrow (\psi \vee l))) \\ \boxed{*}\,(D' \vee (\varphi' \Rightarrow (\chi \vee \neg l))) \end{array}}{\boxed{*}\,(D \vee D' \vee \neg(\varphi \leftrightarrow \varphi') \vee (\varphi \Rightarrow (\psi \vee \chi)))}$$

[R-RES-⇒-2]
$$\frac{\begin{array}{c} \boxed{*}\,(D \vee (\varphi \Rightarrow (\psi \vee l))) \\ \boxed{*}\,(D' \vee \neg l) \end{array}}{\boxed{*}\,(D \vee (\varphi \Rightarrow (\psi \vee D')))}$$

where D' is a literal clause

[L-RES-⇏]
$$\frac{\begin{array}{c} \boxed{*}\,(D \vee (\varphi \Rightarrow \mathbf{false})) \\ \boxed{*}\,(D' \vee (\varphi' \not\Rightarrow \psi)) \end{array}}{\boxed{*}\,(D \vee D' \vee \neg(\neg\varphi \leftrightarrow \varphi'))}$$

[R-RES-⇏-1]
$$\frac{\begin{array}{c} \boxed{*}\,(D \vee (\varphi \Rightarrow (\psi \vee l))) \\ \boxed{*}\,(D' \vee (\varphi' \not\Rightarrow (\chi \vee \neg l))) \end{array}}{\boxed{*}\,(D \vee D' \vee \neg(\neg\varphi \leftrightarrow \varphi') \vee (\varphi' \not\Rightarrow (\psi \vee \chi)))}$$

[R-RES-⇏-2]
$$\frac{\begin{array}{c} \boxed{*}\,(D \vee (\varphi \not\Rightarrow (\psi \vee l))) \\ \boxed{*}\,(D' \vee \neg l) \end{array}}{\boxed{*}\,(D \vee (\varphi \not\Rightarrow (\psi \vee D')))}$$

where D' is a literal clause

Table 2. Inference Rules

The inference rules in Table 2 are presented in simplified form, as some of their conclusions are not transformed into the normal form. For the inference rules [R-RES-⇒-1], [R-RES-⇏-1], and [L-RES-⇏] the resolvent should be rewritten into the normal form. In these cases, distribution can be used to avoid further renaming: a formula as $D \vee (\varphi \vee (\psi \wedge \chi))$ can be rewritten as the clauses $(D \vee \varphi \vee \psi)$ and $(D \vee \varphi \vee \chi)$, for a disjunction D, and formulae φ, ψ, and χ. However, for the resolvents of [REF-1] and [REF-2], further renaming may need to be applied. For instance, if $(\varphi \vee \psi) \not\Rightarrow \chi$ is a subformula in the clause set, then an application of [REF-2] would generate $(\neg\varphi \wedge \neg\psi) \Rightarrow (\neg\varphi \wedge \neg\psi)$. However, as from the definition of the normal form, conjunctions are not allowed on the left-hand side of conditional clauses. Instead of $(\neg\varphi \wedge \neg\psi) \Rightarrow (\neg\varphi \wedge \neg\psi)$, the clauses corresponding to $t \Rightarrow t$ and $t \leftrightarrow (\neg\varphi \wedge \neg\psi)$, where t is a new propositional symbol, are introduced in the clause set. This is not problematic from the point of view of termination, as [REF-1] and [REF-2] are only applied to formulae which can possibly occur in the clause set. As we show later, because the number of such formulae is finite, so it is the number of new propositional symbols that can be introduced as a result of the application of either inference rule.

The soundness of all inference rules follows almost immediately from the axiomatisation of S, as shown in Section 4. The following is the formal definition of a derivation.

Definition 1. *Let Φ be a set of clauses. A derivation in* RES_S *for Φ is a sequence of clause sets $\Phi_0, \Phi_1, \ldots$ where $\Phi_0 = \Phi$ and, for each $i > 0$, $\Phi_{i+1} = \Phi_i \cup \{D\}$, where D is* *the conclusion obtained from Φ_i by an application of one of the inference rules given in Table 2 to premises in Φ_i. We require that D is in simplified form, $D \notin \Phi_i$, and that D is not a propositional tautology.*

Note that the inference rules [REF-1] and [REF-2] introduce tautologies of the form $\varphi \Rightarrow \varphi$, where φ or $\neg\varphi$ occurs on the left-hand side of (negated) conditionals. Those tautologies are needed for completeness. Thus, the constraint for including the resolvent D into the clause set is restricted to classical tautologies, that is, of the form $\varphi \vee \neg\varphi$, for a formula φ.

Definition 2. *Let Φ be a set of clauses. A refutation in* $\mathsf{RES_S}$ *for Φ is a finite derivation $\Phi_0, \Phi_1, \dots, \Phi_k$, $k \in \mathbb{N}$, where* **false** *in Φ_k. We write $\Phi \vdash_{\mathsf{RES_S}}$* **false***, if there is a refutation from Φ in* $\mathsf{RES_S}$.

Definition 3. *Let Φ be a set of clauses. We say that Φ is* saturated *if any further application of the inference rules given in Table 2 to clauses in Φ only generates a clause already in Φ.*

As derivations require progress, a saturated set is a point where a derivation cannot progress any further.

Definition 4. *Let Φ be a set of clauses. A derivation $\Phi_0, \Phi_1, \dots$ in* $\mathsf{RES_S}$ *for Φ* terminates *if there is $k \in \mathbb{N}$ such that Φ_k is saturated or* **false** $\in \Phi_k$.

Before showing the correctness results concerning our calculus, we present an example of a refutation involving a validity with nested conditionals.

Example 1. We show that $\varphi = ((a \Rightarrow b) \wedge (a \Rightarrow c) \wedge (d \Rightarrow c)) \Rightarrow (((a \wedge b) \vee d) \Rightarrow c)$ is a valid formula in S. The negation normal form of the $\neg\varphi$ is $((\neg a \not\Rightarrow \neg b) \vee (\neg a \not\Rightarrow \neg c) \vee (\neg d \not\Rightarrow \neg c)) \not\Rightarrow (((\neg a \vee \neg b) \wedge \neg d) \not\Rightarrow \neg c)$. Clauses 1 to 6 correspond to the normal form of $\neg\varphi$, noting that Clauses 4 to 6 only show the side of the definitions of the propositional symbols introduced by renaming that are needed in the proof.

1. t_0	4. $\boxed{*}(t_2 \vee (a \Rightarrow c))$
2. $\boxed{*}(\neg t_0 \vee ((t_1 \vee t_2 \vee t_3) \not\Rightarrow t_4))$	5. $\boxed{*}(t_3 \vee (d \Rightarrow c))$
3. $\boxed{*}(t_1 \vee (a \Rightarrow b))$	6. $\boxed{*}(\neg t_4 \vee (((\neg a \vee \neg b) \wedge \neg d) \not\Rightarrow \neg c))$

The following refutation follows from the above set of clauses:

7.	$\boxed{*}(\neg t_4 \vee ((\neg a \vee \neg b) \not\Rightarrow \neg c) \vee (\neg d \not\Rightarrow \neg c))$	[L-AND-2,6]
8.	$\boxed{*}(t_3 \vee \neg t_4 \vee ((\neg a \vee \neg b) \not\Rightarrow \neg c))$	[R-RES-$\not\Rightarrow$-1,7,5]
9.	$\boxed{*}(t_3 \vee \neg t_4 \vee (\neg a \not\Rightarrow \neg b) \vee (\neg a \not\Rightarrow \neg c))$	[L-OR-2,8]
10.	$\boxed{*}(t_2 \vee t_3 \vee \neg t_4 \vee (\neg a \not\Rightarrow \neg b))$	[R-RES-$\not\Rightarrow$-1,9,4]
11.	$\boxed{*}(t_1 \vee t_2 \vee t_3 \vee \neg t_4)$	[SIMP-1,R-RES-$\not\Rightarrow$-1,10,3]
12.	$\boxed{*}(t_5 \Rightarrow t_5)$	[REF-2,2, where $t_5 \leftrightarrow \neg(t_1 \vee t_2 \vee t_3)$]
13.	$\boxed{*}(t_5 \vee t_1 \vee t_2 \vee t_3)$	[REF-2,2]
14.	$\boxed{*}(\neg t_5 \vee \neg t_1)$	[REF-2,2]
15.	$\boxed{*}(\neg t_5 \vee \neg t_2)$	[REF-2,2]
16.	$\boxed{*}(\neg t_5 \vee \neg t_3)$	[REF-2,2]
17.	$\boxed{*}(\neg t_5 \vee t_2 \vee t_3 \vee \neg t_4)$	[RES,11,14]
18.	$\boxed{*}(\neg t_5 \vee t_3 \vee \neg t_4)$	[RES,17,15]

19. $\boxed{*}(\neg t_5 \vee \neg t_4)$	[RES,18,16]
20. $\boxed{*}(\neg t_0 \vee ((t_1 \vee t_2 \vee t_3) \not\Rightarrow \neg t_5))$	[R-RES-$\not\Rightarrow$-2,19,2]
21. $\boxed{*}(\neg t_0 \vee \neg(\neg t_5 \leftrightarrow (t_1 \vee t_2 \vee t_3)))$	[SIMP-1,R-RES-$\not\Rightarrow$-1,20,12]
22. $\boxed{*}\neg t_0$	[RES,21,13,14,15,16]
23. **false**	[I-RES-2,22,1]

We note that Clause 8 (resp. Clauses 10 and 11) is in simplified form, as $\mathsf{nnf}(\neg(\neg d \leftrightarrow \neg d))$ (resp. $\mathsf{nnf}(\neg(\neg a \leftrightarrow \neg a))$) simplifies to **false**. The justification of Clause 22 abbreviates several applications of the inference rule [RES] between Clause 21 and the clauses corresponding to the definition of t_5, i.e. Clauses 13 to 16.

4 Correctness

In this section we provide the proofs that RES_S is a sound, complete, and terminating calculus for S. First, we show that given a formula φ, the transformation into CNF is satisfiability preserving.

Theorem 1. *Let φ be a formula in WFF_S. Then, φ is satisfiable if, and only if, $t_0 \wedge \tau(\boxed{*}(t_0 \to \varphi))$ is satisfiable.*

Proof (sketch). The proof is very standard. We first show that φ is satisfiable if, and only if, $t_0 \wedge \boxed{*}(t_0 \to \varphi)$ is satisfiable, as the evaluation of φ does not depend on the evaluation of t_0 and that the operator $\boxed{*}$ does not occur in φ. Then, we show that each of the transformation rules is satisfiability preserving, that is, a formula of the form $\boxed{*}(t \to \varphi')$ is satisfiable if, and only if, $\tau(\boxed{*}(t \to \varphi'))$ is satisfiable. Rewriting is justified by equivalences. For transformation steps which require renaming, let ψ be the subformula of φ' which is being renamed by the transformation function and t' a new propositional symbol. Given the satisfiability of $\boxed{*}(t \to \varphi')$, then there is a model $M = (W, w_0, \pi, R)$ such that $\langle M, w\rangle \models (t \to \varphi')$, for all $w \in W$. We then build a model $M' = (W, w_0, \pi', R)$, which is exactly as M except by the evaluation function. We define $\pi'(w)(p) = \pi(w)(p)$ for all worlds $w \in W$ and propositional symbols p, such that $p \neq t'$; and $\pi'(w)(t') = \mathit{true}$ if, and only if, $\langle M, w\rangle \models \psi$. We then show that, for all worlds $w \in W$, we have that $\langle M', w\rangle \models (t \to \varphi'[\psi \mapsto t']) \wedge (t' \leftrightarrow \psi)$, where $\varphi'[\psi \mapsto t']$ is the result of replacing some occurrences of the subformula ψ in φ' by t'. The if part follows easily by taking into account that, by construction, t' and ψ are satisfied at the same worlds in a model; hence $(t \to \varphi'[t' \mapsto \psi])$ is satisfiable in all worlds $w \in W$. It follows that $\tau(\boxed{*}(t \to \varphi'))$ is satisfiable. Finally, by induction on the number of steps of a transformation, we obtain that φ and $t_0 \wedge \tau(\boxed{*}(t_0 \to \varphi))$ are equisatisfiable. □

We note that the transformation into the normal form results in a formula which is polynomial in the size of the original formula, as the number of subformulae of a formula is linear in the size of the original formula and also because the renaming procedure introduces at most two copies of every subformula (plus a constant number of connectives). The procedure is also terminating, as only complex subformulae of a formulae are either rewritten or renamed.

Lemma 2. *Let Φ be a set of clauses. Then, any derivation in RES_S from Φ terminates.*

Proof. Let P^+_Φ be the set of propositional symbols occurring in Φ. We define $\mathsf{P}^-_\Phi = \{\neg p \mid p \in \mathsf{P}_\Phi\}$ and $\mathsf{L}_\Phi = \mathsf{P}^+_\Phi \cup \mathsf{P}^-_\Phi$. Let Sub^+_Φ be the set of all propositional subformulae occurring in Φ, $Sub^-_\Phi = \{\mathsf{nnf}(\neg\varphi) \mid \varphi \in Sub^+_\Phi\}$, and $Sub^\pm_\Phi = Sub^+_\Phi \cup Sub^-_\Phi$. As only propositional formulae can occur on the left-hand side of the conditional implications and its dual, then the number of additional literals that might be introduced during the application of [REF-1] and [REF-2] is bounded by $|Sub^\pm_\Phi|$. As P_Φ and $Sub^\pm_\Phi$ are finite, so it is $\mathscr{P}(\mathsf{L}_\Phi \cup Sub^\pm_\Phi \bigcup_{\varphi \in Sub^\pm_\Phi}\{t_\varphi \Rightarrow t_\varphi, t \leftrightarrow \varphi\})$, where t_φ is a new propositional symbol. Let C_Φ be the largest set of clauses that can be constructed from L_Φ, $Sub^\pm_\Phi$, and the conditionals introduced by [REF-1] and [REF-2] together with the double-implications introduced for renaming of formulae in $Sub^\pm_\Phi$. From Definition 1, for all derivations $\Phi_0, \Phi_1, \ldots$ from Φ, we have that $\Phi_i \subset C_\Phi$ and also that $\Phi_i \subset \Phi_{i+1}$, for all $i > 0$. Thus, every derivation must terminate. $\square$

For soundness of $\mathsf{RES_S}$, we need to show that, for each of the inference rules given in Table 2, if the premises of the inference rules are satisfiable, so it is their conclusion. We omit most of the easy cases, but note that soundness of [L-AND-2] and [L-OR-2] follow almost immediately from the contrapositive forms of the axioms **A5** and **A4**, that is, $((\varphi \wedge \psi) \not\Rightarrow \chi) \rightarrow (\varphi \not\Rightarrow \chi) \vee (\psi \not\Rightarrow \chi))$ and $((\varphi \vee \psi) \not\Rightarrow \chi) \rightarrow (\varphi \not\Rightarrow \psi) \vee (\varphi \not\Rightarrow \chi))$, respectively. The inference rules [REF-1] and [REF-2] are obviously sound, as they introduce instances of the axiom **A1** into the clause set. It is also very easy to see that [I-RES-1], [I-RES-2], and [RES] are only variations of the classical binary resolution: the fact they are sound follows also almost immediately from the results in [18]. The next lemmas show the soundness of [R-RES-$\not\Rightarrow$-1] and [L-RES-$\not\Rightarrow$].

Lemma 3. *Let Φ be a set of clauses with $\{\boxed{*}(D \vee (\varphi \Rightarrow (\psi \vee l))), \boxed{*}(D' \vee (\varphi' \not\Rightarrow (\chi \vee \neg l)))\} \subseteq \Phi$. If Φ is satisfiable, then $\Phi \cup \{\boxed{*}(D \vee D' \vee \neg(\neg\varphi \leftrightarrow \varphi') \vee (\varphi' \not\Rightarrow \psi \vee \chi)\}$ is satisfiable.*

Proof. If Φ is satisfiable, as $\{\boxed{*}(D \vee (\varphi \Rightarrow (\psi \vee l))), \boxed{*}(D' \vee (\varphi' \not\Rightarrow (\chi \vee \neg l)))\} \subseteq \Phi$, from the definition of satisfiability of sets, there is a model $M = (W, w_0, \pi, R)$ such that (1) $\langle M, w_0\rangle \models \boxed{*}(D \vee (\varphi \Rightarrow (\psi \vee l)))$ and (2) $\langle M, w_0\rangle \models \boxed{*}(D' \vee (\varphi' \not\Rightarrow (\chi \vee \neg l)))$. From (1) and the semantics of the universal operator, for all $w \in W$, we have that (3) $\langle M, w\rangle \models (D \vee (\varphi \Rightarrow (\psi \vee l)))$. Analogously, from (2), for all $w \in W$, we have that (4) $\langle M, w\rangle \models (D' \vee (\varphi' \not\Rightarrow (\chi \vee \neg l)))$. Let w be any world in W. From (3) and (4), by distribution, there are four cases: (i) $\langle M, w\rangle \models (D \wedge D')$; (ii) $\langle M, w\rangle \models (D \wedge (\varphi' \not\Rightarrow (\chi \vee \neg l))$; (iii) $\langle M, w\rangle \models (D' \wedge (\varphi \Rightarrow (\psi \vee l))$; or (iv) $\langle M, w\rangle \models (\varphi \Rightarrow (\psi \vee l)) \wedge (\varphi' \not\Rightarrow (\chi \vee \neg l))$. It is easy to see that if Cases (i), (ii), or (iii) hold, then we have that (5) $\langle M, w\rangle \models (D \vee D')$. For the fourth case, there are two possibilities: either (6) $\langle M, w\rangle \models \neg(\neg\varphi \leftrightarrow \varphi')$; or (7) $\langle M, w\rangle \models (\neg\varphi \leftrightarrow \varphi')$. From (7) and from the fact that $\langle M, w\rangle \models (\varphi' \not\Rightarrow (\chi \vee \neg l))$, by soundness of [RPE], we obtain that (8) $\langle M, w\rangle \models (\neg\varphi \not\Rightarrow (\chi \vee \neg l))$. From (8), the fact that $\langle M, w\rangle \models (\varphi \Rightarrow (\psi \vee l))$, the semantics of conjunctions, and the soundness of **A2**, by the soundness of [MP], we obtain that $\langle M, w\rangle \models (\neg\varphi \not\Rightarrow (\psi \vee l) \wedge (\chi \vee \neg l))$. By the soundness of resolution, applied on the right-hand side of the preferential conditional, we obtain that (9) $\langle M, w\rangle \models (\neg\varphi \not\Rightarrow (\psi \vee \chi))$. From (9) and (7), we obtain that (10) $\langle M, w\rangle \models (\varphi' \not\Rightarrow (\psi \vee \chi))$. From (5), (6), (10), and from the semantics of disjunction, we finally have that $\langle M, w\rangle \models (D \vee D' \vee \neg(\neg\varphi \leftrightarrow \varphi') \vee (\varphi' \not\Rightarrow \psi \vee \chi))$.

As this holds for any world w, from the semantics of the universal operator, it follows that $\langle M, w_0\rangle \models \boxed{*}(D \vee D' \vee \neg(\neg\varphi \leftrightarrow \varphi') \vee (\varphi' \not\Rightarrow \psi \vee \chi))$. We conclude that $\Phi \cup \{\boxed{*}(D \vee D' \vee \neg(\neg\varphi \leftrightarrow \varphi') \vee (\varphi' \not\Rightarrow \psi \vee \chi))\}$ is satisfiable. $\square$

Lemma 4. *Let Φ be a set of clauses with $\{\boxed{*}(D \vee (\varphi \Rightarrow \textbf{false})), \boxed{*}(D' \vee (\varphi' \not\Rightarrow \psi))\} \subseteq \Phi$. If Φ is satisfiable, then $\Phi \cup \{\boxed{*}(D \vee D' \vee \neg(\neg\varphi \leftrightarrow \varphi'))\}$ is satisfiable.*

Proof (Sketch). The proof follows from the fact that $\boxed{*}(D \vee (\varphi \Rightarrow \textbf{false}))$ is semantically equivalent to $\boxed{*}(D \vee (\varphi \Rightarrow \psi \wedge \neg\psi))$ and, from **A3**, this implies that $\boxed{*}(D \vee (\varphi \Rightarrow \neg\psi))$ is satisfiable. By Lemma 3 taking Φ with $\{\boxed{*}(D \vee (\varphi \Rightarrow \neg\psi)), \boxed{*}(D' \vee (\varphi' \not\Rightarrow \psi))\} \subseteq \Phi$, together with the soundness of [SIMP-1], we obtain that $\Phi \cup \{\boxed{*}(D \vee D' \vee \neg(\neg\varphi \leftrightarrow \varphi'))\}$ is satisfiable. $\square$

The proof that [R-RES-$\Rightarrow$-1] is sound is pretty similar to that of Lemma 3. Soundness of [R-RES-$\Rightarrow$-2] and [R-RES-$\not\Rightarrow$-2] follow easily from the fact that the right-hand side of the operators $\Rightarrow$ and $\not\Rightarrow$ are monotonic. Thus resolution can be applied on the the right-hand side of modal formulae, taking into account that the other premise is also in the scope of the universal operator. The next theorem shows that $\mathsf{RES_S}$ is sound, the proof of which follows from our argumentation, as above, and Lemmas 3 and 4.

Theorem 2. *Let Φ be a set of clauses and $\Phi_0, \Phi_1, \ldots$ be a derivation in $\mathsf{RES_S}$ for Φ. If Φ is satisfiable, then every Φ_i, $i \geq 0$, is satisfiable.*

The soundness proof, given above, shows that if Φ is satisfiable, then there is no refutation from Φ, that is: if there is a structure M such that $M \models \Phi$, then $\Phi \not\vdash_{\mathsf{RES_S}} \textbf{false}$. In the following, we prove the completeness of $\mathsf{RES_S}$: if $\Phi \not\vdash_{\mathsf{RES_S}} \textbf{false}$, then there is a structure M such that $M \models \Phi$. The proof follows the standard construction of canonical models for modal logics and is heavily based on that given in [6].

Given a set of clauses Φ, we construct a structure (W, S), where W is a set (of worlds) and S is a binary relation over W, as follows. Let I and G denote the set of initial and global clauses in Φ, respectively. Let $G' = \{\varphi \mid \boxed{*}\,\varphi \in G\}$. Let φ_I, φ_G, and $\varphi_{G'}$ denote the conjunction of formulae in I, G and G', respectively. Let $Cl(\Phi)$ be the closure of Φ under subformulae and simple negation. That is, $Cl(\Phi)$ is the least set such that:

- $\varphi_I \wedge \varphi_G \wedge \varphi_{G'} \in Cl(\Phi)$;
- If $\varphi \in Cl(\Phi)$ and $\varphi' \in \mathsf{Subf}(\varphi)$, then $\varphi' \in Cl(\Phi)$;
- If $\varphi \in Cl(\Phi)$ and then $\mathsf{nnf}(\neg\varphi) \in Cl(\Phi)$;

where $\mathsf{Subf}(\varphi)$ denotes the set of subformulae of φ. (Recall that we consider sub-conjunctions of subformulae as subformulae.) Let $\mathscr{A}, \mathscr{B} \in \mathscr{P}(Cl(\Phi))$ be sets of formulae in the powerset of the closure of Φ. A set of formulae $\mathscr{A}$ is $\mathsf{RES_S}$-consistent if, and only if, (i) for all $\varphi \in \mathscr{A}$, $\neg\boxed{*}\,\varphi \notin \mathscr{A}$; and (ii) $\mathscr{A} \not\vdash_{\mathsf{RES_S}} \textbf{false}$. A consistent set of formulae $\mathscr{A}$ is maximal with respect to $Cl(\Phi)$ if, and only if, (i) $G \subseteq \mathscr{A}$ (in order to ensure that all global clauses are in all sets); and (ii) there is no consistent $\mathscr{B} \in \mathscr{P}(Cl(\Phi))$ such that $\mathscr{A} \subset \mathscr{B}$. Although there is no specific inference rules for dealing with formulae of the form $\neg\boxed{*}\,\varphi$, as they do not occur in the normal form, a set containing such a formula cannot be maximal consistent, as G is a subset of all maximal sets.

An *atom* is a maximal consistent set in $\mathscr{P}(Cl(\Phi))$, the powerset of $Cl(\Phi)$. Let Atoms_{Φ} be the set of all atoms constructed from Φ. In the following, we denote atoms by a,b,c,d, and set of atoms by A,B,C,D. For an atom a, we write $\bigwedge a$ (resp. $\bigvee a$) as an abbreviation for the conjunction (resp. disjunction) of the formulae in a. A *world* is defined as a pair (a,A), where a is an atom and A a set of atoms. Given two atoms a and b, and a set of atoms B, we define that $\mathsf{Prefer}(a,b,B)$ holds if, and only if, $\bigwedge a \wedge \neg((\bigwedge b \vee \bigvee \bigwedge_{b' \in B} b') \Rightarrow \bigvee \bigwedge_{b' \in B} b')$ is $\mathsf{RES_S}$-consistent. We define the structure $M = (W, w_0, \pi, R)$ as follows:

- $W = \{(a,A) \mid a \in Atoms_{\Phi}, A \subseteq (Atoms_{\Phi} \setminus \{a\})\}$;
- $w_0 = (a,\emptyset) \in W$, with $\Phi \subseteq a$;
- For all propositional symbols $p \in \mathsf{P}$, let $\pi((a,A))(p) = true$ if, and only if, $p \in a$;
- For all worlds $(a,A) \in W$, let $W_{(a,A)} = \{(b,B) \in W \mid \mathsf{Prefer}(a,b,B)\}$ and set $(b,B) <_{(a,A)} (c,C)$, if $C \cup \{c\} \subseteq B$. For all worlds $w', w'' \in W_w$, set $\leq_w$ such that, $w' \leq_w w'$; and, if $w' <_w w''$, then $w' \leq_w w''$.

Intuitively, a in (a,A) is a world satisfying $\bigwedge a$ which is strictly preferred to all worlds in A. The evaluation function assigns truth values to propositional symbols according to their value in a. The set $W_{(a,A)}$ contains all worlds (b,B) such that it is consistent with a that b is strictly preferred to worlds satisfying atoms in B. Note that the construction of $W_{(a,A)}$ depends on the set of conditionals in a, as defined by the predicate $\mathsf{Prefer}(\cdot)$. Thus, if two atoms a and b share the same set of conditional formulae, then $W_{(a,A)}$ and $W_{(b,A')}$ are exactly the same. It is easy to check that the relation $\leq_w$ is indeed a preorder. Given those definitions, we establish the completeness of $\mathsf{RES_S}$. First, we note that the construction of a model from a saturated set of clauses is closed under the inference rules of $\mathsf{RES_S}$ and also that the following two properties hold.

Lemma 5. *Let Φ be a saturated set of clauses, G be the set of global clauses in Φ, $Cl(\Phi)$ be the closure of Φ, and a be an atom in Atoms_{Φ}, the set of all atoms constructed from Φ. For any formula $\varphi \in Cl(\Phi) \cup G$, $\varphi \in a$ if, and only if, $\mathsf{nnf}(\neg\varphi) \notin a$.*

Lemma 6. *Let Φ be a saturated set of clauses, G be the set of global clauses in Φ, and a be an atom in Atoms_{Φ}, the set of all atoms constructed from Φ. For any formula $\boxed{*}\,\varphi \in G$, $\boxed{*}\,\varphi \in a$ if, and only if, $\varphi \in a$.*

The proof of the truth lemma depends on the following two results. For $\varphi \Rightarrow \psi \in (a,A)$, as an additional (induction) hypotheses, we assume that for all subformulae φ' of $\varphi \Rightarrow \psi$ and all worlds (a',A'), we have that $\varphi' \in a'$ if, and only if, $\langle M, (a',A') \rangle \models \varphi'$. For the purpose of a contradiction, assume that $\langle M, (a,A) \rangle \models \varphi \Rightarrow \psi$.

Lemma 7. *Let Φ be a saturated set of clauses, $M = (W, w_0, \pi, R)$ be the structure constructed as above for Φ, $(a,A) \in W$ be a world in M, and φ, ψ be formulae in $Cl(\Phi)$. If $\varphi \Rightarrow \psi \in a$ in (a,A) then $\langle M, (a,A) \rangle \models \varphi \Rightarrow \psi$.*

Proof. For the purpose of contradiction, assume $\varphi \Rightarrow \psi \in (a,A)$, but that $\langle M, (a,A) \rangle \not\models \varphi \Rightarrow \psi$. If $\langle M, (a,A) \rangle \not\models \varphi \Rightarrow \psi$, then, from the semantics of $\Rightarrow$, there is a world $(b,B) \in W_{(a,A)}$ such that (b,B) is a minimal φ-world and $\langle M, (b,B) \rangle \models \varphi \wedge \neg\psi$. It follows that (i) $\langle M, (b,B) \rangle \models \varphi$ and (ii) $\langle M, (b,B) \rangle \models \neg\psi$. By induction hypotheses, from (i), we have that $\varphi \in b$ and, from (ii), that $\psi \notin b$ (or, equivalently, that $\mathsf{nnf}(\neg\psi) \in b$). If $(b,B) \in W_{(a,A)}$, then, from the definition of $W_{(a,A)}$, we have that $\mathsf{Prefer}(a,b,B)$ holds, that is, $a \wedge \neg((b \vee \bigvee B) \Rightarrow \bigvee B)$ is $\mathsf{RES_S}$-consistent. However, we can show that for $\varphi, \neg\psi \in b$, we have that $\mathsf{Prefer}(a,b,B)$ and $\varphi \Rightarrow \psi \in (a,A)$ is not $\mathsf{RES_S}$-consistent (see Appendix A for the detailed proof), which contradicts with having $(b,B) \in W_{(a,A)}$. Thus, it cannot be the case that $\langle M,(a,A)\rangle \not\models \varphi \Rightarrow \psi$. Hence, $\langle M,(a,A)\rangle \models \varphi \Rightarrow \psi$. □

Lemma 8. *Let Φ be a saturated set of clauses, $M = (W, w_0, \pi, R)$ be the structure constructed as above for Φ, $(a,A) \in W$ be a world in M, and φ, ψ be formulae in $Cl(\Phi)$. If $\langle M,(a,A)\rangle \models \varphi \Rightarrow \psi$, then $\varphi \Rightarrow \psi \in a$ in (a,A).*

Proof. We show the contrapositive, i.e. if $\varphi \Rightarrow \psi \notin a$ in (a,A), then $\langle M,(a,A)\rangle \not\models \varphi \Rightarrow \psi$. If $\varphi \Rightarrow \psi \notin a$, then $a \wedge (\varphi \Rightarrow \psi)$ is not $\mathsf{RES_S}$-consistent. By Lemma 5, $\mathsf{nnf}((\neg\varphi \not\Rightarrow \neg\psi)) \in (a,A)$. For the purpose of contradiction, assume that $\langle M,(a,A)\rangle \models \varphi \Rightarrow \psi$. Thus, for all (b,B) in $W_{(a,A)}$ such that (b,B) is a minimal φ-world, $\langle M,(b,B)\rangle \models \varphi \wedge \psi$. It follows that $\langle M,(b,B)\rangle \models \varphi$ and $\langle M,(b,B)\rangle \models \psi$. By inductive hypothesis, $\varphi \in (b,B)$ and $\psi \in (b,B)$. As $(b,B) \in W_{(a,A)}$, from the definition of $W_{(a,A)}$, $\mathsf{Prefer}(a,b,B)$ holds, i.e. $a \wedge \neg((b \vee \bigvee B) \Rightarrow \bigvee B)$ is $\mathsf{RES_S}$-consistent. We can show however that, for $\varphi, \psi \in b$, we have that $\neg((b \vee \bigvee B) \Rightarrow \bigvee B)$ and $\neg(\varphi \Rightarrow \psi)$ is not $\mathsf{RES_S}$-consistent (see Appendix A for the detailed proof). Thus, it cannot be the case that $\langle M,(a,A)\rangle \models \varphi \Rightarrow \psi$. Hence, $\langle M,(a,A)\rangle \not\models \varphi \Rightarrow \psi$. □

Lemma 9. *Let Φ be a saturated set of clauses, $M = (W, w_0, \pi, R)$ be the structure constructed as above for Φ, $(a,A) \in W$ be a world in M, and φ a formula in $Cl(\Phi)$. Then, $\varphi \in a$ in (a,A) if, and only if, $\langle M,(a,A)\rangle \models \varphi$.*

The proofs for the classical connectives make use of Lemmas 5 and 6 and is routine. For formulae of the form $(\varphi \Rightarrow \psi)$, the proof follows from Lemmas 7 and 8. Completeness of $\mathsf{RES_S}$ follows immediately from the truth lemma (Lemma 9), as stated above.

5 Discussion and Further Work

We have presented a sound and complete resolution calculus for Burgess' system S. Our main motivation is to present a purely syntactic calculus that is both easy and efficient to implement. The only other calculus for S we are aware of is that of [19] which heavily relies on semantic arguments for the definition of proof rules, and is therefore non-trivial to both implement and optimise. In contrast, the resolution system here is purely given in syntactic terms. The design of the resolution rules, while not generated from the axioms by means of an algorithmic procedure, closely follows the axiomatisation. A different axiomatisation would lead to a different set of inference rules, in particular those related to propagation ([L-AND-2], [L-OR-2], [REF-1] and [REF-2]). The main technical challenge was the completeness proof, for which we have adapted a canonical model construction to the resolution setting, obtaining a direct proof (without translating to other calculi) where the main obstacle in the proof was to integrate the construction with pre-processing of formulae into normal form.

To fully substantiate our claim regarding ease of implementation and efficiency, we plan to implement and compare both our calculus and that of [19].

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A Proofs

The next two proofs were automatically generated by a prototype prover which implements the calculus given in this paper. Only clauses needed in the refutation are shown. Also the inference rule [SIMP-1] is always applied together with [R-RES-$\not\Rightarrow$-1], so clauses are already in simplified form. First, as part of the proof of Lemma 7, we show that for $\varphi, \neg\psi \in b$, we have that $\mathsf{Prefer}(a,b,B)$ and $\varphi \Rightarrow \psi \in (a,A)$ is contradictory.

1.	t_1	[Assumption]
2.	$(\neg t_1 \vee (\varphi \Rightarrow \psi))$	[Assumption, $\varphi \Rightarrow \psi \in a$]
3.	$(\neg t_1 \vee (((\neg\varphi \vee \psi) \wedge \neg B) \not\Rightarrow \neg B))$	[Assumption, $\varphi, \neg\psi \in b$, $(b,B) \in W_{(a,A)}$]
4.	$(\neg t_1 \vee (\neg B \not\Rightarrow \neg B) \vee ((\neg\varphi \vee \psi) \not\Rightarrow \neg B)$	[L-AND-2,3]
8.	$(B \vee \neg t_3 \vee t_2)$	[REF-2,3]
9.	$(\neg t_2 \vee t_3)$	[REF-2,3]
11.	$(\varphi \vee \neg t_2)$	[REF-2,3]
12.	$(\neg\psi \vee \neg t_2)$	[REF-2,3]
14.	$((B \Rightarrow B))$	[REF-2,4]
15.	$((t_2 \Rightarrow t_2))$	[REF-2,4]
25.	$(B \vee \neg\psi \vee \neg t_3)$	[RES,8,12,t_2]
26.	$(B \vee \varphi \vee \neg t_3)$	[RES,8,11,t_2]
38.	$(\neg B \vee \neg t_4)$	[R-RES-$\not\Rightarrow$-1,3,14]
39.	$(B \vee \neg t_4 \vee t_2)$	[R-RES-$\not\Rightarrow$-1,3,14]
40.	$(B \vee t_4 \vee t_5)$	[R-RES-$\not\Rightarrow$-1,3,14]
41.	$(\psi \vee \neg\varphi \vee \neg t_5)$	[R-RES-$\not\Rightarrow$-1,3,14]
42.	$(\neg B \vee \neg t_5)$	[R-RES-$\not\Rightarrow$-1,3,14]
44.	$(\neg t_1 \vee t_4)$	[R-RES-$\not\Rightarrow$-1,3,14,B]
49.	$(\varphi \vee \neg t_3 \vee \neg t_5)$	[RES,42,26,B]
50.	$(\neg\psi \vee \neg t_3 \vee \neg t_5)$	[RES,42,25,B]
254.	$(\psi \vee \neg t_3 \vee \neg t_5)$	[RES,41,49,φ]
276.	$(\neg t_3 \vee \neg t_5)$	[RES,254,50,ψ]
292.	$(\neg t_2 \vee \neg t_5)$	[RES,276,9,$\neg t_3$]
493.	$(B \vee \neg t_2 \vee t_4)$	[RES,40,292,t_5]
540.	$(B \vee \neg\psi \vee \neg t_4)$	[RES,39,12,t_2]
547.	$(\neg t_4 \vee t_2)$	[RES,39,38,B]
557.	$(\varphi \vee \neg t_4)$	[RES,547,11,t_2]
558.	$(\neg t_4 \vee t_3)$	[RES,547,9,t_2]
559.	$(\neg t_1 \vee t_2)$	[RES,547,44,$\neg t_4$]
560.	$(\neg t_4 \vee \neg t_5)$	[RES,547,292,t_2]
814.	$(\neg t_1 \vee (((\neg\varphi \vee \psi) \wedge \neg B) \not\Rightarrow \neg\psi \vee \neg t_4))$	[R-RES-$\not\Rightarrow$-2,540,3,B]
834.	$(\neg\varphi \vee \neg t_6 \vee t_5)$	[R-RES-$\not\Rightarrow$-1,814,2]
837.	$(B \vee \neg t_7 \vee t_2)$	[R-RES-$\not\Rightarrow$-1,814,2]
838.	$(\varphi \vee \neg t_7)$	[R-RES-$\not\Rightarrow$-1,814,2]
839.	$(\neg\varphi \vee \neg t_3 \vee t_7)$	[R-RES-$\not\Rightarrow$-1,814,2]
1333.	$(\neg\varphi \vee \neg t_4 \vee t_7)$	[RES,839,558,$\neg t_3$]
1361.	$(\neg t_4 \vee t_7)$	[RES,1333,557,φ]
1372.	$(\neg t_1 \vee t_7)$	[RES,1361,44,$\neg t_4$]
1374.	$(B \vee \neg t_2 \vee t_7)$	[RES,1361,493,$\neg t_4$]
1885.	$(B \vee \neg t_7 \vee t_4)$	[RES,837,493,t_2]
1911.	$(\neg t_5 \vee \neg t_7 \vee t_4)$	[RES,1885,42,B]

1981. $(\neg t_5 \lor \neg t_7)$	[RES,1911,560,t_4]
2915. $(\neg\varphi \lor \neg t_6 \lor \neg t_7)$	[RES,834,1981,t_5]
2942. $(\neg t_6 \lor \neg t_7)$	[RES,2915,838,φ]
2984. $(\neg t_1 \lor \neg t_6)$	[RES,2942,1372,$\neg t_7$]
2985. $(B \lor \neg t_2 \lor \neg t_6)$	[RES,2942,1374,$\neg t_7$]
3123. $(\neg t_1 \lor (((\neg\varphi \lor \psi) \land \neg B) \not\Rightarrow \neg t_2 \lor \neg t_6))$	[R-RES-$\not\Rightarrow$-2,2985,3,B]
3901. $(\neg t_2 \lor \neg t_9 \lor t_5)$	[R-RES-$\not\Rightarrow$-1,3123,15]
3904. $(B \lor \neg t_{10} \lor t_2)$	[R-RES-$\not\Rightarrow$-1,3123,15]
3906. $(\neg t_2 \lor t_{10})$	[R-RES-$\not\Rightarrow$-1,3123,15]
6854. $(\neg t_1 \lor (((\neg\varphi \lor \psi) \land \neg B) \not\Rightarrow \neg t_{10} \lor t_2))$	[R-RES-$\not\Rightarrow$-2,3904,3,B]
10225. $(\neg t_2 \lor \neg t_9)$	[RES,3901,292,t_5]
10314. $(\neg t_1 \lor \neg t_9)$	[RES,10225,559,$\neg t_2$]
22010. $(\neg t_1 \lor (((\neg\varphi \lor \psi) \land \neg B) \not\Rightarrow \neg t_{10} \lor \neg\psi))$	[R-RES-$\not\Rightarrow$-2,6854,12,t_2]
23844. $(\neg t_1 \lor (((\neg\varphi \lor \psi) \land \neg B) \not\Rightarrow \neg\psi \lor \neg t_2))$	[R-RES-$\not\Rightarrow$-2,22010,3906,$\neg t_{10}$]
23923. $(\neg t_1 \lor t_6 \lor (((\neg\varphi \lor \psi) \land \neg B) \not\Rightarrow \neg t_2))$	[R-RES-$\not\Rightarrow$-1,23844,2,$\neg\psi$]
24001. $(\neg t_1 \lor t_6 \lor t_9)$	[R-RES-$\not\Rightarrow$-1,23923,15,$\neg t_2$]
24126. $(\neg t_1 \lor t_6)$	[RES,24001,10314,t_9]
24194. $(\neg t_1)$	[RES,24126,2984,t_6]
24242. **false**	[I-RES-2,1,24194]

The following refutation is part of the proof of Lemma 8, where we show that, for $\varphi, \psi \in b$, we have that $\neg((b \lor \bigvee B) \Rightarrow \bigvee B)$ and $\neg(\varphi \Rightarrow \psi) \in a$ is not $\mathsf{RES_S}$-consistent

1. t_1	
2. $(\neg t_1 \lor (((\neg\varphi \lor \neg\psi) \land \neg B) \not\Rightarrow \neg B))$	[Assumption, $b \in W_{(a,A)}$]
3. $(\neg t_1 \lor (\neg\varphi \not\Rightarrow \neg\psi))$	[Assumption, $\neg(\varphi \Rightarrow \psi) \in a$]
4. $(\neg t_1 \lor (\neg B \not\Rightarrow \neg B) \lor ((\neg\varphi \lor \neg\psi) \not\Rightarrow \neg B))$	[L-AND-2,2]
11. $(\varphi \lor \neg t_2)$	[REF-2,2]
12. $(\psi \lor \neg t_2)$	[REF-2,2]
13. $(\neg\varphi \lor \neg\psi \lor t_2)$	[REF-2,2]
15. $(B \Rightarrow B)$	[REF-2,4]
16. $(t_2 \Rightarrow t_2)$	[REF-2,4]
44. $(\neg t_1 \lor (\neg\varphi \not\Rightarrow \neg t_2))$	[R-RES-$\not\Rightarrow$-2,3,12,$\neg\psi$]
55. $(\neg\varphi \lor \neg t_2 \lor \neg t_6)$	[R-RES-$\not\Rightarrow$-1,44,16]
56. $(\varphi \lor \neg t_6 \lor t_2)$	[R-RES-$\not\Rightarrow$-1,44,16]
64. $(\neg t_1 \lor t_6)$	[R-RES-$\not\Rightarrow$-1,44,16,$\neg t_2$]
318. $(\neg\psi \lor \neg t_6 \lor t_2)$	[RES,56,13,φ]
578. $(\neg t_2 \lor \neg t_6)$	[RES,55,11,$\neg\varphi$]
627. $(\neg\psi \lor \neg t_6)$	[RES,578,318,$\neg t_2$]
630. $(\neg\psi \lor \neg t_1)$	[RES,627,64,$\neg t_6$]
2580. $(\neg B \lor \neg t_9 \lor t_{10})$	[R-RES-$\not\Rightarrow$-1,2,15]
2582. $(\neg B \lor \neg t_{10})$	[R-RES-$\not\Rightarrow$-1,2,15]
2584. $(B \lor \neg t_9 \lor t_2)$	[R-RES-$\not\Rightarrow$-1,2,15]
2592. $(\neg t_1 \lor t_9)$	[R-RES-$\not\Rightarrow$-1,2,15,$\neg B$]
3769. $(B \lor \psi \lor \neg t_9)$	[RES,2584,12,t_2]
11984. $(\neg B \lor \neg t_9)$	[RES,2580,2582,t_{10}]
12128. $(\psi \lor \neg t_9)$	[RES,11984,3769,$\neg B$]
12165. $(\psi \lor \neg t_1)$	[RES,12128,2592,$\neg t_9$]
12205. $\neg t_1$	[RES,12165,630,ψ]
12230. **false**	[I-RES-2,1,12205,t_1]