Towards the Semantics of QBF Clauses*

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Extended Abstract

The ongoing interest in the problem of Quantified Boolean Formulas (QBF) has resulted in numerous solving techniques, e.g. [22, 19, 12, 23, 13], as well as various resolution-based, clausal calculi [21, 29, 3, 20, 7] which advance our understanding of the techniques and formalise the involved reasoning.

While a substantial progress in terms of understanding these calculi has already been made on the front of proof complexity [3, 20, 7–9, 5, 10, 15, 26, 18, 17], the question of semantics of the involved intermediate clauses has until now received comparatively less attention. In many cases, the semantics is left only implicit, determined by the way in which the clauses are allowed to interact via inferences. This is in stark contrast with propositional or first-order logic, in which a clause can always be identified with the set of its models.

In my talk, I would like to expand on why I find this situation unsatisfying, give examples of what I thought a uniform underlying semantics of QBF clauses *could* be, and, as a teaser for our talk at SAT, briefly explain what I and Bernhard Gleiss finally identified as a viable candidate [28].

The Mystery of QBF Tautologies. One hint that something is not quite right in the way we understand QBF clauses can be demonstrated on the treatment of tautologies. A tautology is a clause which contains both a literal and its complement. While in the setting of propositional and first-order logic tautologies are *harmless* (in the sense that they are always vacuously satisfied and thus can be safely added or discarded), in the study of resolution-based calculi for QBF we encounter tautologies which can be *harmful* (generation of tautologies is explicitly prohibited in Q-Res [21], because they would make the calculus unsound), but also *useful* (the long-distance resolution calculus LD-Q-Res [30, 1] gains exponential power over Q-Res by allowing generation of certain tautologies). How does one resolve this discrepancy? Shouldn't we give up on treating a QBF clause as a disjunction of its literals?

Semantics and Soundness. One of most important properties of a calculus is its soundness and one of the most common methods for showing soundness is relating the inferences of the calculus and the semantics of the manipulated clauses by a notion of logical *entailment*. We know how to show soundness of

* This work was supported by ERC Starting Grant 2014 SYMCAR 639270 and the Austrian research projects FWF S11403-N23 and S11409-N23.

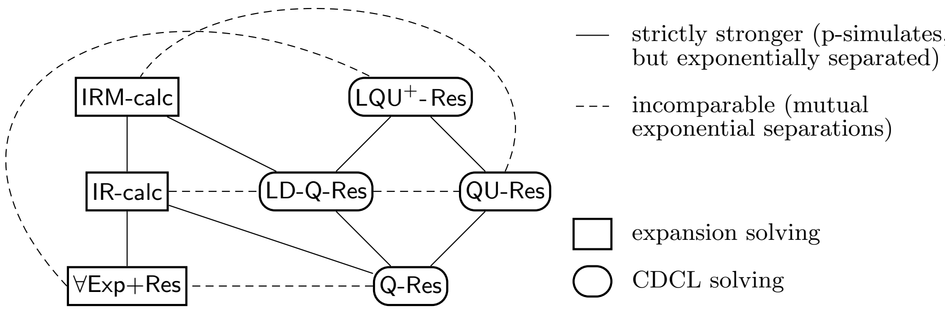

Fig. 1. QBF resolution calculi [8] and their simulation order.

Q-Res using semantical methods [24, 27] and I will argue that this technique is getting implicitly extended to LD-Q-Res via the notion of a *shadow clause* [4] introduced for the purpose of strategy extraction [2].

A different notion of semantics can be provided to the expansion-derived calculi ∀Exp+Res [20] and IR-calc [6] via a translation from QBF to first-order logic [25], and so soundness of these calculi can be established with the help of first-order model theory [16, 11]. Extending this approach to accommodate IRM-calc [6], a calculus which unifies the instantiation flavour inherent to the expansion-derived calculi with the essence of long-distance steps coming from LD-Q-Res (see Fig. 1), was one of the focal points behind this work. I will report on the challenges and lessons the most direct route in this direction provides.

But why should we actually be so interested in semantic methods for showing soundness? The main reasons is that the corresponding argument can be structured *modularly*, treating each inferences rule in separation and concluding by trivial induction along the refutation: "Since every conclusion of a rule is entailed by its premises and since the empty clause cannot have a model, the input axioms cannot have a model either." In this sense, a semantic method enables the notion of a *sound inference*, an inference that can be added to a calculus without affecting its soundness. In contrast, the currently known proof of soundness of IRM-calc [6] is global, manipulating the whole refutation monolithically under an arguably complex inductive invariant. Should one want to add another rule to IRM-calc, the whole proof might need to be redone from scratch.

Semantics via Strategies. In our paper [28], we propose to use strategies, more specifically, the *partial strategies for the universal player*, as the central objects manipulated within a refutation. We show how strategies arise from the formula matrix and identify operations for obtaining new strategies by combining old ones. We then provide the missing meaning to the intermediate clauses of the existing calculi by seeing them as *abstractions* of these strategies. This way, we obtain soundness of all the calculi from Fig. 1 in a purely local, modular way.

Although primarily viewed as a model-theoretical concept in this context, the strategies also carry the obvious computational aspect. One can see the above mentioned abstraction as providing a specification for a strategy when understood as a program. This relates our approach to the Curry-Howard correspondence: We can treat the specification clause as a type and the derivation which lead to it and for which a strategy is the semantical denotation as the implementing program. The specification of the empty clause can then be read as "my strategy is total and, therefore, winning."

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