Convergence of Simultaneously and Sequentially Unraveled TRSs for Normal Conditional TRSs*

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Abstract

Unravelings, which are transformations of a conditional term rewriting system (CTRS, for short) into an unconditional term rewriting system (TRS, for short), are useful to prove confluence and operational termination of some CTRSs. A simultaneous unraveling has been proposed for normal 1-CTRSs and a sequential one has been proposed for deterministic 3-CTRSs, the class of which includes normal 1-CTRSs. In this paper, we first show that for a normal 1-CTRS, the simultaneously unraveled TRS is orthogonal iff so is the sequentially unraveled one. Then, we show that for a normal 1-CTRS, if the simultaneously unraveled TRS is terminating, then so is the sequentially unraveled one. Finally, we show that for a normal 1-CTRS with termination of the unraveled TRS, the simultaneously unraveled TRS is locally confluent iff so is the sequentially unraveled one.

1 Introduction

Conditional term rewriting [14, Chapter 7] is known to be much more complicated than unconditional term rewriting in the sense of analyzing properties (cf. [12]). A popular approach to the analysis of conditional term rewriting systems (CTRSs, for short) is to transform a CTRS into an unconditional term rewriting system (TRS, for short) that is in general an overapproximation of the CTRS w.r.t. reduction. This approach enables us to use techniques for the analysis of TRSs, which are well investigated in the literature.

Unravelings [9, 10, 13] are useful to prove *confluence* and *operational termination* [8] of CTRSs because of the following results: (a) a deterministic 3-CTRS (3-DCTRS, for short) is confluent if the unraveled TRS is confluent and the CTRS is *weakly left-linear* (WLL, for short) [6, 7], and (b) a 3-DCTRS is operationally terminating if the unraveled TRS is terminating [3]. A *simultaneous unraveling* has been proposed for normal 1-CTRSs [9, 14], and a *sequential unraveling* has been proposed for 3-DCTRSs [10, 13]. Normal 1-CTRSs are 3-DCTRS and both the simultaneous and sequential unravelings are applicable to normal 1-CTRSs. For this reason, to prove confluence and operational termination of normal 1-CTRSs, we can use both the simultaneous and sequential unravelings. For example, CO3 [11], a confluence prover for CTRSs, tries to prove confluence via the simultaneous unraveling, and, if it fails, then uses the sequential one.

In this paper, we first show that for a normal 1-CTRS, the simultaneously unraveled TRS is orthogonal iff so is the sequentially unraveled one (Section 4). Then, we show that for a normal 1-CTRS, if the simultaneously unraveled TRS is terminating, then so is the sequentially unraveled one (Section 5). Finally, we show that for a normal 1-CTRS with termination of the unraveled TRS, the simultaneously unraveled TRS is locally confluent iff so is the sequentially unraveled one (Section 6). The second and third results imply that for a normal 1-CTRS, the simultaneously unraveled TRS is convergent iff so is the sequentially unraveled one.

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2 Preliminaries

We omit basic notions and notations for term rewriting [2, 14], and we assume that the reader is familiar with them. In this section, we briefly recall the notions and notations of CTRSs.

An (oriented) *conditional rewrite rule* over a signature $\mathcal{F}$ is a triple (ℓ, r, c), denoted by $\ell \to r \Leftarrow c$, such that the *left-hand side* ℓ is a non-variable term in $T(\mathcal{F}, \mathcal{V})$, the *right-hand side* r is a term in $T(\mathcal{F}, \mathcal{V})$, and the *conditional part* c is a sequence $s_1 \twoheadrightarrow t_1, \ldots, s_k \twoheadrightarrow t_k$ of term pairs ($k \geq 0$) where all of $s_1, t_1, \ldots, s_k, t_k$ are terms in $T(\mathcal{F}, \mathcal{V})$. In particular, a conditional rewrite rule $\ell \to r \Leftarrow c$ is called *unconditional* if the conditional part c is the empty sequence ϵ, and we may abbreviate it to $\ell \to r$. We sometimes attach a unique label ρ to the conditional rewrite rule $\ell \to r \Leftarrow c$ by denoting $\rho : \ell \to r \Leftarrow c$, and we use the label to refer to the rewrite rule. An *(oriented) conditional term rewriting system* (CTRS, for short) over a signature $\mathcal{F}$ is a set of conditional rules over $\mathcal{F}$, and it is called a *term rewriting system* (TRS, for short) if every rule $\ell \to r \Leftarrow c$ in the system is unconditional and $\mathcal{V}ar(\ell) \supseteq \mathcal{V}ar(r)$.

A CTRS $\mathcal{R}$ is called *normal* if for every rule $\ell \to r \Leftarrow s_1 \twoheadrightarrow t_1, \ldots, s_k \twoheadrightarrow t_k \in \mathcal{R}$, all of $t_1, \ldots, t_k$ are ground normal forms of $\mathcal{R}_u$ where $\mathcal{R}_u = \{\ell \to r \mid \ell \to r \Leftarrow c \in \mathcal{R}\}$. A CTRS $\mathcal{R}$ is called a *1-CTRS* (*3-CTRS*, resp.) if $\mathcal{V}ar(r, c) \subseteq \mathcal{V}ar(\ell)$ ($\mathcal{V}ar(r) \subseteq \mathcal{V}ar(\ell, c)$, resp.) for every rule $\ell \to r \Leftarrow c \in \mathcal{R}$. A CTRS $\mathcal{R}$ is called *deterministic* (DCTRS, for short) if for every rule $\ell \to r \Leftarrow s_1 \twoheadrightarrow t_1, \ldots, s_k \twoheadrightarrow t_k \in \mathcal{R}$, $\mathcal{V}ar(s_i) \subseteq \mathcal{V}ar(\ell, t_1, \ldots, t_{i-1})$ for all $1 \leq i \leq k$.

3 Unravelings

An *unraveling* U is a transformation of CTRSs into TRSs such that for every CTRS $\mathcal{R}$, we have that $\to_{\mathcal{R}} \subseteq \to^*_{U(\mathcal{R})}$ and $U(\mathcal{R} \cup \mathcal{R}') = U(\mathcal{R}) \cup \mathcal{R}'$ for any TRS $\mathcal{R}'$ [9, 12]. For a CTRS $\mathcal{R}$ over a signature $\mathcal{F}$, we denote the extended signature of $\mathcal{F}$ via U by $\mathcal{F}_{U(\mathcal{R})}$. Given a finite set $X = \{o_1, \ldots, o_n\}$ of objects, a sequence $o_1, o_2, \ldots, o_n$ under some arbitrary order on the objects is denoted by $\overrightarrow{X}$.

A *simultaneous unraveling* for normal 1-CTRSs [9] has been refined as follows.

Definition 3.1 ($\mathbb{U}_{sim}$ [14])**.** *Let $\mathcal{R}$ be a normal 1-CTRS over a signature $\mathcal{F}$. For each conditional rule $\rho : \ell \to r \Leftarrow s_1 \twoheadrightarrow t_1, \ldots, s_k \twoheadrightarrow t_k$ in $\mathcal{R}$, we introduce a new function symbol U^ρ, and transform ρ into a set of two unconditional rules as follows:*

$$\mathbb{U}_{sim}(\rho) = \{ \quad \ell \to U^\rho(s_1, \ldots, s_k, \overrightarrow{\mathcal{V}ar(\ell)}), \quad U^\rho(t_1, \ldots, t_k, \overrightarrow{\mathcal{V}ar(\ell)}) \to r \quad \}$$

Note that if $k = 0$, then $\mathbb{U}_{sim}(\ell \to r) = \{\ell \to r\}$. $\mathbb{U}_{sim}$ is straightforwardly extended to normal 1-CTRSs: $\mathbb{U}_{sim}(\mathcal{R}) = \bigcup_{\rho \in \mathcal{R}} \mathbb{U}_{sim}(\rho)$. Note that $\mathbb{U}_{sim}(\mathcal{R})$ is a TRS over $\mathcal{F}_{\mathbb{U}_{sim}(\mathcal{R})}$.

A *sequential unraveling* for 3-DCTRSs [10] has been refined as follows.

Definition 3.2 ($\mathbb{U}_{seq}$ [13, 14])**.** *Let $\mathcal{R}$ be a 3-DCTRS over a signature $\mathcal{F}$. For each conditional rule $\rho : \ell \to r \Leftarrow s_1 \twoheadrightarrow t_1, \ldots, s_k \twoheadrightarrow t_k$ in $\mathcal{R}$, we introduce k new function symbols $U^\rho_1, \ldots, U^\rho_k$, and transform ρ into a set of $k+1$ unconditional rules as follows:*

$$\mathbb{U}_{seq}(\rho) = \{ \quad \ell \to U^\rho_1(s_1, \overrightarrow{X_1}), \quad U^\rho_1(t_1, \overrightarrow{X_1}) \to U^\rho_2(s_2, \overrightarrow{X_2}), \quad \ldots, \quad U^\rho_k(t_k, \overrightarrow{X_k}) \to r \quad \}$$

where $X_i = \mathcal{V}ar(l, t_1, \ldots, t_{i-1})$ for $1 \leq i \leq k$. Note that if $k = 0$, then $\mathbb{U}_{seq}(\ell \to r) = \{\ell \to r\}$. $\mathbb{U}_{seq}$ is straightforwardly extended to DCTRSs: $\mathbb{U}_{seq}(\mathcal{R}) = \bigcup_{\rho \in \mathcal{R}} \mathbb{U}_{seq}(\rho)$. Note that $\mathbb{U}_{seq}(\mathcal{R})$ is a TRS over $\mathcal{F}_{\mathbb{U}_{seq}(\mathcal{R})}$.

Example 3.3. *Consider the following normal 1-CTRS [4] (278.trs in Cops[1]):*

$$\mathcal{R}_1 = \left\{ \begin{array}{l} \mathsf{proc}(y,m) \to \mathsf{proc}(\mathsf{app}(\mathsf{map}(\mathsf{self},\mathsf{nil}),\mathsf{split}_2(m,y)),m) \\ \qquad \Leftarrow \mathsf{leq}(m,\mathsf{len}(y)) \twoheadrightarrow \mathsf{true}, \mathsf{e}(\mathsf{split}_1(m,y)) \twoheadrightarrow \mathsf{false}, \\ \mathsf{proc}(y,m) \to \mathsf{proc}(\mathsf{split}_2(m,\mathsf{app}(\mathsf{map}(\mathsf{self},\mathsf{nil}),y)),m) \\ \qquad \Leftarrow \mathsf{leq}(m,\mathsf{len}(y)) \twoheadrightarrow \mathsf{false}, \mathsf{e}(\mathsf{split}_1(m,\mathsf{app}(\mathsf{map}(\mathsf{self},\mathsf{nil}),y))) \twoheadrightarrow \mathsf{false} \end{array} \right\} \cup \mathcal{R}_2$$

where $\mathcal{R}_2$ is a TRS defining app, map, split$_2$, leq, split$_1$, *and* e *as in [4, pp. 42–43]. $\mathcal{R}_1$ is unraveled by $\mathbb{U}_{sim}$ and $\mathbb{U}_{seq}$ as follows:*

$$\mathbb{U}_{sim}(\mathcal{R}_1) = \left\{ \begin{array}{r l} \mathsf{proc}(y,m) & \to U_1(\mathsf{leq}(m,\mathsf{len}(y)),\mathsf{e}(\mathsf{split}_1(m,y)),y,m), \\ U_1(\mathsf{true},\mathsf{false},y,m) & \to \mathsf{proc}(\mathsf{app}(\mathsf{map}(\mathsf{self},\mathsf{nil}),\mathsf{split}_2(m,y)),m), \\ \mathsf{proc}(y,m) & \to U_2(\mathsf{leq}(m,\mathsf{len}(y)),\mathsf{e}(\mathsf{split}_1(m,\mathsf{app}(\mathsf{map}(\mathsf{self},\mathsf{nil}),y))),y,m), \\ U_2(\mathsf{false},\mathsf{false},y,m) & \to \mathsf{proc}(\mathsf{split}_2(m,\mathsf{app}(\mathsf{map}(\mathsf{self},\mathsf{nil}),y)),m) \end{array} \right\} \cup \mathcal{R}_2$$

$$\mathbb{U}_{seq}(\mathcal{R}_1) = \left\{ \begin{array}{r l} \mathsf{proc}(y,m) & \to U_3(\mathsf{leq}(m,\mathsf{len}(y)),y,m), \\ U_3(\mathsf{true},y,m) & \to U_4(\mathsf{e}(\mathsf{split}_1(m,y)),y,m), \\ U_4(\mathsf{false},y,m) & \to \mathsf{proc}(\mathsf{app}(\mathsf{map}(\mathsf{self},\mathsf{nil}),\mathsf{split}_2(m,y)),m), \\ \mathsf{proc}(y,m) & \to U_5(\mathsf{leq}(m,\mathsf{len}(y)),y,m), \\ U_5(\mathsf{false},y,m) & \to U_6(\mathsf{e}(\mathsf{split}_1(m,\mathsf{app}(\mathsf{map}(\mathsf{self},\mathsf{nil}),y))),y,m), \\ U_6(\mathsf{false},y,m) & \to \mathsf{proc}(\mathsf{split}_2(m,\mathsf{app}(\mathsf{map}(\mathsf{self},\mathsf{nil}),y)),m) \end{array} \right\} \cup \mathcal{R}_2$$

4 Orthogonality of Unraveled TRSs

In this section, we show that for a normal 1-CTRS $\mathcal{R}$, $\mathbb{U}_{sim}(\mathcal{R})$ is orthogonal (i.e., left-linear and non-overlapping) iff so is $\mathbb{U}_{seq}(\mathcal{R})$.

Let $\mathcal{R}$ be a normal 1-CTRS over a signature $\mathcal{F}$. By definition, for a rule $\ell \to r \in \mathbb{U}_{sim}(\mathcal{R})$, the left-hand side ℓ is either in $T(\mathcal{F},\mathcal{V})$ or of the form $U^\rho(t_1,\ldots,t_k,x_1,\ldots,x_n)$ where $t_1,\ldots,t_k$ are ground normal forms of $\mathcal{R}_u$. In the latter case, the rule $\ell \to r$ is not overlapping with any rule in $\mathbb{U}_{sim}(\mathcal{R})$. For this reason, by definition, if we have two overlapping rules $\ell_1 \to r_1, \ell_2 \to r_2 \in \mathbb{U}_{sim}(\mathcal{R})$, then we have two overlapping rules $\ell_1 \to r_1', \ell_2 \to r_2' \in \mathbb{U}_{seq}(\mathcal{R})$.

Lemma 4.1. *For a normal 1-CTRS $\mathcal{R}$, $\mathbb{U}_{sim}(\mathcal{R})$ is non-overlapping iff so is $\mathbb{U}_{seq}(\mathcal{R})$.*

It follows from the definition of $\mathbb{U}_{sim}$ and [12, Theorem 3.9 (1)] that $\mathbb{U}_{sim}(\mathcal{R})$ is left-linear iff so is $\mathbb{U}_{seq}(\mathcal{R})$. Therefore, the following theorem is a direct consequence of Lemma 4.1.

Theorem 4.2. *For a normal 1-CTRS $\mathcal{R}$, $\mathbb{U}_{sim}(\mathcal{R})$ is orthogonal iff so is $\mathbb{U}_{seq}(\mathcal{R})$.*

Since orthogonality is decidable, given a normal 1-CTRS $\mathcal{R}$, if we prove confluence of an unraveled TRS ($\mathbb{U}_{sim}(\mathcal{R})$ or $\mathbb{U}_{seq}(\mathcal{R})$) via orthogonality, then we can also prove confluence of the other unraveled TRS.

5 Termination of Unraveled TRSs

In this section, we show that for a normal 1-CTRS $\mathcal{R}$, if $\mathbb{U}_{sim}(\mathcal{R})$ is terminating, then so is $\mathbb{U}_{seq}(\mathcal{R})$.

It is shown in [12] that for a normal 1-CTRS $\mathcal{R}$ over a signature $\mathcal{F}$, there exists a *tree homomorphism* $\phi_{\mathcal{R}}$ such that for all terms $s,t \in T(\mathcal{F}_{\mathbb{U}_{seq}(\mathcal{R})},\mathcal{V})$, if $s \to^*_{\mathbb{U}_{seq}(\mathcal{R})} t$, then

[1] http://cops.uibk.ac.at

Table 1: the result of proving termination of the unraveled TRSs from Cops.

result	AProVE		NaTT		CO3	
	$\mathbb{U}_{sim}(\cdot)$	$\mathbb{U}_{seq}(\cdot)$	$\mathbb{U}_{sim}(\cdot)$	$\mathbb{U}_{seq}(\cdot)$	$\mathbb{U}_{sim}(\cdot)$	$\mathbb{U}_{seq}(\cdot)$
YES	39	39	35	35	24	25
NO	9	9	9	9	—	—
MAYBE	0	0	7	7	27	26
timeout (300 seconds)	3	3	0	0	0	0

$\phi_{\mathcal{R}}(s) \to^*_{\mathbb{U}_{sim}(\mathcal{R})} \phi_{\mathcal{R}}(t)$. The tree homomorphism can be extended for *dependency pairs* [1] so that for all terms $s,t \in T(\mathcal{F}_{\mathbb{U}_{seq}(\mathcal{R})}, \mathcal{V})$, if $s^\sharp \to_{DP(\mathbb{U}_{seq}(\mathcal{R}))} t^\sharp$, then $(\phi_{\mathcal{R}}(s))^\sharp\ (= \cup \to_{DP(\mathbb{U}_{sim}(\mathcal{R}))})\ (\phi_{\mathcal{R}}(t))^\sharp$, where $DP(\mathcal{R}')$ denotes the set of dependency pairs of a TRS $\mathcal{R}'$ and $u^\sharp$ denotes the term obtained from u by replacing the root symbol by the corresponding *marked* symbol. This implies that if $s,t \in T(\mathcal{F},\mathcal{V})$ and $s^\sharp\ (\to^*_{\mathbb{U}_{seq}(\mathcal{R})} \cdot \to_{DP(\mathbb{U}_{seq}(\mathcal{R}))} \cdot \to^*_{\mathbb{U}_{seq}(\mathcal{R})})^+\ t^\sharp$, then $(\phi_{\mathcal{R}}(s))^\sharp\ (\to^*_{\mathbb{U}_{sim}(\mathcal{R})} \cdot \to_{DP(\mathbb{U}_{sim}(\mathcal{R}))} \cdot \to^*_{\mathbb{U}_{sim}(\mathcal{R})})^+\ (\phi_{\mathcal{R}}(t))^\sharp$. Thus, an infinite *dependency chain* of $\mathbb{U}_{seq}(\mathcal{R})$ can be converted to an infinite dependency chain of $\mathbb{U}_{sim}(\mathcal{R})$.

Theorem 5.1. *For a normal 1-CTRS $\mathcal{R}$, if $\mathbb{U}_{sim}(\mathcal{R})$ is terminating, then so is $\mathbb{U}_{seq}(\mathcal{R})$.*

The converse of Theorem 5.1 does not hold in general. For example, for $\mathcal{R}_2 = \{\ \mathsf{a} \to \mathsf{b} \Leftarrow \mathsf{c} \twoheadrightarrow \mathsf{d}, \mathsf{a} \twoheadrightarrow \mathsf{e}\ \}$, $\mathbb{U}_{seq}(\mathcal{R}_2)$ is terminating but $\mathbb{U}_{sim}(\mathcal{R}_2)$ is not.

Since termination is undecidable, unlike orthogonality, Theorem 5.1 does not imply that (a) if we have proved termination of $\mathbb{U}_{sim}(\mathcal{R})$ using some method, then we could directly prove termination of $\mathbb{U}_{seq}(\mathcal{R})$ using some method that does not rely on Theorem 5.1. It is not easy to prove (a) for all existing methods to prove termination of TRSs. Instead of proving (a), we examined (a) for 51 normal 1-CTRSs in Cops.[1] Our experiments were performed on OS X 10.11.6 equipped with an Intel Core i5 CPU at 2.9 GHz with 8 GB RAM, and we used AProVE [5], NaTT [15], and CO3 [11] as termination provers. Table 1 illustrates the number of benchmarks for each result, and indicates that the results for $\mathbb{U}_{sim}$ and $\mathbb{U}_{seq}$ are almost the same—the methods implemented in CO3 are very simple, and thus, the number of YES for $\mathbb{U}_{sim}$ and $\mathbb{U}_{seq}$ are slightly different.

6 Local Confluence of Unraveled TRSs

In this section, we show that for a normal 1-CTRS $\mathcal{R}$ with termination of the unraveled TRSs $\mathbb{U}_{sim}(\mathcal{R})$ and $\mathbb{U}_{seq}(\mathcal{R})$, if $\mathbb{U}_{sim}(\mathcal{R})$ is locally confluent (i.e., confluent), then so is $\mathbb{U}_{seq}(\mathcal{R})$.

Let $\mathcal{R}$ be a normal 1-CTRS over a signature $\mathcal{F}$. As described in Section 4, every overlap of the unraveled TRS is caused by two rules $\ell_1 \to r_1, \ell_2 \to r_2$ such that $\ell_1, \ell_2 \in T(\mathcal{F},\mathcal{V})$. The tree homomorphism $\phi_{\mathcal{R}}$ in Section 5 can be used for joinability of critical pairs of $\mathbb{U}_{sim}(\mathcal{R})$ from joinability of $\mathbb{U}_{seq}(\mathcal{R})$, and vice versa.

Theorem 6.1. *Let $\mathcal{R}$ be a normal 1-CTRS such that $\mathbb{U}_{sim}(\mathcal{R})$ is terminating. $\mathbb{U}_{sim}(\mathcal{R})$ is locally confluent iff so is $\mathbb{U}_{seq}(\mathcal{R})$.*

For terminating TRSs, (local) confluence is decidable (see [2, p. 140]). Therefore, given a normal 1-CTRS $\mathcal{R}$, if we prove termination of $\mathbb{U}_{sim}(\mathcal{R})$ or $\mathbb{U}_{seq}(\mathcal{R})$, and if we prove local confluence of an unraveled TRS ($\mathbb{U}_{sim}(\mathcal{R})$ or $\mathbb{U}_{seq}(\mathcal{R})$), then we can also prove local confluence of the other unraveled TRS.

7 Conclusion

In this paper, we showed that for a normal 1-CTRS, (1) the simultaneously unraveled TRS is orthogonal iff so is the sequentially unraveled one, (2) if the simultaneously unraveled TRS is terminating, then so is the sequentially unraveled one, and (3) under termination of the unraveled TRS, the simultaneously unraveled TRS is locally confluent iff so is the sequentially unraveled one. The second and third results imply that for a normal 1-CTRS, the simultaneously unraveled TRS is convergent iff so is the sequentially unraveled one. If $\mathcal{R}$ is WLL and $\mathbb{U}_{sim}(\mathcal{R})$ or $\mathbb{U}_{seq}(\mathcal{R})$ is confluent, then $\mathcal{R}$ is confluent [6, 7]. Therefore, to prove confluence of a WLL normal 1-CTRS by either orthogonality of the unraveled TRS or termination and joinability of critical pairs of the unraveled TRS, there is no difference between the use of $\mathbb{U}_{sim}$ and $\mathbb{U}_{seq}$, except for the power of a termination prover we use (see Table 1).

The sequential unraveling has been improved to preserve confluence of CTRSs as much as possible [7, $\mathbb{U}_{conf}$]. We will adapt the results in this paper to the improved sequential unraveling and then we will consider the efficiency of proving confluence via CO3. In addition, we will compare the simultaneous and sequential unravelings w.r.t. other confluence criteria for TRSs.

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